0.1 Exponential and Log Equations

A common use of logarithms and exponents is in solving equations. Often, the definition of the logarithm is sufficient to solve equations.

$$\log_b c = a \longleftrightarrow b^a = c$$

Example Solve: $\log_x 5 = -2$

This is equivalent to $x^{-2} = 5$, then inverting both sides leads to $x^2 = \frac{1}{5}$, and taking square root we have $x = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$.

The usual rule of equations is that if the left side equals the right side, then the same operation to both sides of the equation results in another equation. Naturally, logarithms are defined only for positive numbers.

$$\log_b x = \log_b y \iff x = y \iff b^x = b^y$$

Recall also that the exponential and log functions with the same base are inverses of eachother.

$$\begin{aligned} b^{\log_b x} &= x \\ \log_b b^x &= x \end{aligned}$$

Solving Common Exponential Equations

One of the most common exponential equations to solve is of the form

$$a = be^{cx}$$

where a, b, and c are constants and x is the variable in question.

Here, taking the natural log of both sides will save the day.

Example Solve $12 = 5e^{3t}$

Take the natural log of both sides, then solve for t.

$$\begin{aligned} 12 &= 5e^{3t} \\ \ln 12 &= \ln\left(3^{3t}\right) \\ 2.4849 &= 3t \\ t &= \frac{2.4849}{3} = .8283 \end{aligned}$$

Example Solve $3^{4.2t} = 6^{t+1}$

There is no convenient log to use (the only convenient ones are the common and natural logs), so we will use the common log here. Note that it was convenient to use the natural log in the example above because $\ln e^{3t} = 3t$.

$$\begin{aligned} 3^{4.2t} &= 6^{t+1} \\ \log 3^{4.2t} &= \log 6^{t+1} \\ 4.2t \log 3 &= (t+1)\log 6 \\ 4.2t(.4771) &= (t+1)(.7782) \\ 2.0038t &= .7782t + .7782 \\ -.7782t & \quad -.7782t \\ 1.2256t &= .7782 \\ t &= \frac{.7782}{1.2256} = .6350 \end{aligned}$$

Check: $3^{4.2(.635)} = 3^{2.667} = 18.73$
$6^{.6350+1} = 6^{1.6350} = 18.72$

Exercises

Solve the following equations for t.

1. $3.5 = e^t$
2. $4.7 = 10^t$
3. $2.3 = e^{2t}$
4. $14.7 = 10^{t+2}$
5. $34.5 = 1.2e^{3t}$
6. $50.2 = e^{2.45t}$
7. $12.6 = 1.2e^{-0.03t}$
8. $0.0023 = e^{-0.025t}$