0.1 Logarithms

The logarithm function (or simply log function) is the inverse function for the exponential function.

$$\log_b x = y \qquad \longleftrightarrow \qquad b^y = x$$

If we want to know what $\log_2 8$ is, we need to think about what power to raise 2 in order to get 8: $2^{\square} = 8$. Since $2^3 = 8$, we then know that $\log_2 8 = 3$.

$\log_5 5 = 1$because $5^1 = 5$
$\log_{10} \sqrt{10} = \frac{1}{2}$ because $10^{\frac{1}{2}} = \sqrt{10}$
$\log_{10} 10^{32} = 32$ because $10^{32} = 10^{32}$
$\log_{16} 2 = \frac{1}{4}$ because $16^{1/4} = 2$
$\log_3 \frac{1}{9} = -2$ because $3^{-2} = \frac{1}{9}$
$\log_6 1 = 0$ because $6^0 = 1$
$\log_{1/2} 8 = -3$ because $\left(\frac{1}{2}\right)^{-3} = 8$
$\log_{10} \frac{1}{\sqrt{10}} = -\frac{1}{2}$because $10^{-1/2} = \frac{1}{\sqrt{10}}$

Computing logs

We can use the definition to find $\log_2 32$ because we see that $2^5 = 32$, and so $\log_2 32 = 5$.

What about $\log_{10} 5.68$? We need the power x to raise 10 so that $10^x = 5.86$.

Scientific calculators have two logarithm buttons, log for base 10 and ln for base e.

Definition $\log y \equiv \log_{10} y \qquad \ln y \equiv \log_e y$

These are called the common log and natural log functions respectively.

Example Find $\log_{10} 5.68$

This means $\log 5.68$, and it is found using the [log] button on your calculator.
$\log 5.68 = .7543$
There are often many digits available using a calculator. It is traditional to use 4 significant digits, and you should follow this tradition here.
To check, $10^{.7543} = 5.67937$. Here we showed 6 digits from the calculator result, not the same as 5.68, but it does round to 5.68 using 3 significant digits. When using approximations, you should get used to such mild loss of accuracy as it is unavoidable.

Example Find $\log_e 112$

This is the natural log, and $\log_e 112 = \ln 112$. Using a calculator's [ln] button, we find that $\ln 112 = 4.718$. To check, $e^{4.718} = 112$.

Example Find $\log 2.35 \times 10^{23}$

There is an elegant property of logarithms to simplify this calculation in the next section, but for now...
On your calculator, you need to be able to enter 2.35×10^{23} in scientific notation. On the TI 83+ calculator, you use the [EE] button.
[log] 2.35 [EE] 23 [Enter]
This appears as $\log 2.35E23$, and the result is about 23.3711
If you do not have such a scientific notation button, but you do have a log button, you might note that $\log 2.35 = .3711$, and the power of ten is 23, then $\log 2.35 \times 10^{23} = 23.3711$.
In general, $\log a \times 10^n = n + \log a$

Logarithms of Other Bases

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$$

We can calculate with base 10 (common log) and base e (natural log), and this rule tells us that we can calculate the logarithm with any base using common or natural logs.

Example Find $\log_5 61.2$

$$\log_5 61.2 = \frac{\log 61.2}{\log 5} = \frac{1.7868}{.6990} = 2.5562$$

In practice, most calculators will allow you to do the calculation direction as in $\log(61.2)/\log(5) = 2.5563$
The difference in the last digit results from rounding using different steps, but such approximations are usually adequate.

Example Find $\log_2 .00015$

$$\begin{aligned}\log_2 .00015 &= \frac{\ln .00015}{\ln 2} \\ &= \frac{-8.8049}{.6931} = -12.7037\end{aligned}$$

Check: $2^{-12.7037} = .0001499$, which is not exact but close enough given approximations made.

Exercises

Calculate the following.

1. $\log_2 64$
2. $\log_5 125$
3. $\log_{10} 1000000$
4. $\log_3 \left(\frac{1}{9}\right)$
5. $\log_7 1$
6. $\log_6 6$
7. $\log_{10} 10^7$
8. $\log_e e^3$
9. $\log 4.75$
10. $\log 123.7$
11. $\log 1.23 \times 10^{34}$
12. $\log 5.72 \times 10^{-12}$
13. $\ln 34.2$
14. $\ln 0.0026$
15. $\log_3 16.7$
16. $\log_{16} 300$
17. $\log_{.25} 9.12$
18. $\log_9 97.21$
19. $\log_7 \sqrt[3]{49}$
20. $\log \sqrt{5}$