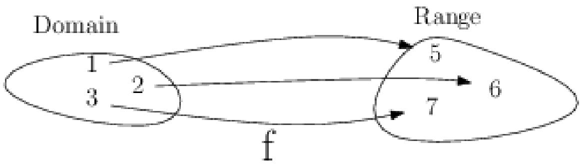
0.1 1-1 and Inverse Functions



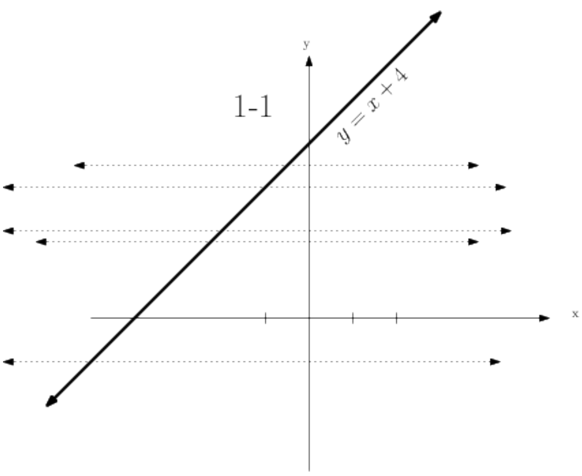
A one to one (1 to 1) function has a unique element in the domain for each element in the range. In the picture above, we see that 5 came from 1, or $f(1) = 5$. When we can identify a unique element in the domain for each element in the domain, we can define an inverse function to "bring us back". In this case, $f^{-1}(5) = 1$, and $f^{-1}(7) = 3$.

This picture above could be a partial representation of the function $f(x) = x + 4$. In words, when we "f" a number, the result is four added to the number. We can undo this function easily by subtracting four from an element of the range, or $f^{-1}(x) = x - 4$.

Definition: A 1-1 function f has an inverse function f^{-1} so that

$$f^{-1}(f(x)) = x = f(f^{-1}(x))$$

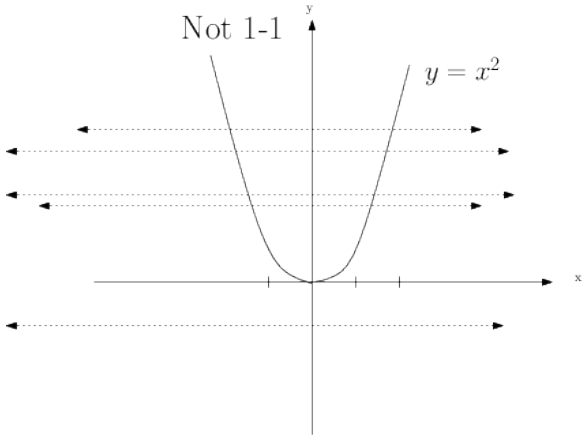
The graph of a function f can be shown by graphing $y = f(x)$. For the function $f(x) = x + 4$, we graph $y = x + 4$.



Note that the dotted horizontal lines intersect the graph of $y = x + 4$ in at most one point. This is an alternate definition of a 1-1 function.

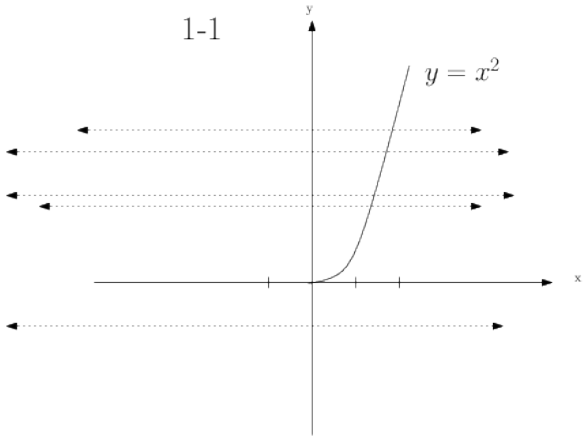
Definition: The graph of a 1-1 function intersects a horizontal line in at most one point.

Some functions are not 1-1, like $f(x) = x^2$.



Note that there are horizontal lines which intersect the graph of $y = x^2$ in more than one point.

We can only find an inverse function for 1-1 functions. However, sometimes we can "adjust" a function which is not 1-1 to create a 1-1 function. Suppose we restrict the domain of $f(x) = x^2$ to have the domain $[0, \infty)$.



This function is 1-1, and it has the inverse function $f^{-1}(x) = \sqrt{x}$.

There is a algebraic technique to finding the inverse function $f^{-1}(x)$ of a given function $f(x)$.
1) Replace f(x) with y.
2) Switch x and y. That is turn x into y, and y into x.
3) Solve for y.
4) Replace y with $f^{-1}(x)$.

Example: Find the inverse function for $f(x) = 3x - 2$
1) y = 3x - 2
2) x = 3y - 2
3) 3y - 2 = x
3y = x + 2
$y = \dfrac{x+2}{3}$
4) $f^{-1}(x) = \dfrac{x+2}{3}$.

Example: Find the inverse function for $g(x) = 5x^3 - 2$

1) $y = 5x^3 - 2$
2) $x = 5y^3 - 2$
3) $5y^3 - 2 = x$
$5y^3 = x + 2$
$y^3 = \dfrac{x+2}{5}$
$y = \sqrt[3]{\dfrac{x+2}{5}}$
4) $g^{-1}(x) = \sqrt[3]{\dfrac{x+2}{3}}$

Exercises

1. Find a function that is not 1-1, graph it, and find two values a and b so that f(a) = f(b).
2. f(x) = 3x + 7 . Find $f^{-1}(x)$
3. g(x) = 2x + 7. Find $g^{-1}(x)$
4. $f(x) = 3x$. Find $f^{-1}(x)$.
5. $h(x) = x - 8$. Find $h^{-1}(x)$
6. $f(x) = \dfrac{-1}{3}x - 8$. Find $f^{-1}(x)$
7. $f(x) = \sqrt[3]{2x+1}$. Find $f^{-1}(x)$
8. $j(x) = 2x^3 + 1$. Find $j^{-1}(x)$.