1 Functions

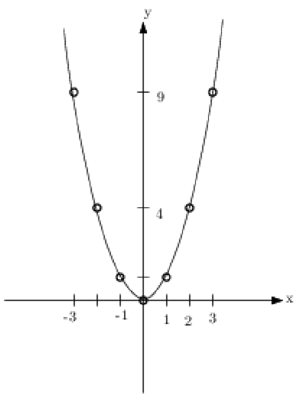
1.1 Quadratic Functions

The quadratic function

$$f(x) = ax^2 + bx + c$$

occurs in a variety of applications. The shape of the curve, when graphed, is a parabola which has interesting geometrical properties.

The graph below of the function $f(x) = x^2$ is sketched by assigning f(x) to the vertical coordinate y then sketching $y = x^2$. This practice of assigning the function value to y for the purpose of making a graph is usual and often unmentioned.



x	$y = x^2$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

There are a couple of interesting features of a quadratic function.

Zeros of $f(x) = ax^2 + bx + c$

The zeros of a quadratic function are the x-axis intercepts. These are found by solving the equation

$$0 = ax^2 + bx + c$$

Any available method can be used. The quadratic formula always works, yet sometimes it is easier to solve by factoring or taking roots.

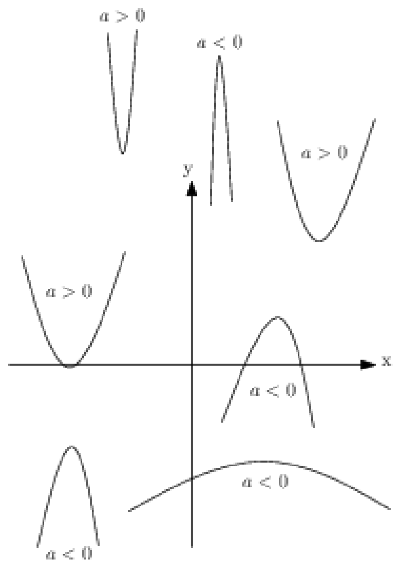
Recal the quadratic formula for the zeros: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Y-axis Intercept of $f(x) = ax^2 + bx + c$

This is the coordinate $(0, f(0))$ on the y-axis. Set x = 0, then f(0) is the y-coordinate of the y-axis intercept.

Shape: Cup Up or Cup Down of $f(x) = ax^2 + bx + c$

If a $>$ 0, the graph is cup up. If a $<$ 0, the graph is cup down.



Vertex of $f(x) = ax^2 + bx + c$

In this general form of the parabola, the vertex is

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

Note the quadratic formula for the zeros, which expanded is

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

If the zeros are real numbers, $\frac{-b}{2a}$ is exactly between the zeros. If there is one real root (when $\sqrt{b^2 - 4ac} = 0$), then the zero is $\frac{-b}{2a}$. If there are no real zeros, then $\frac{-b}{2a}$ is the real component of the complex zeros. In practice, it is usually easiest to find the vertex by calculating $\frac{-b}{2a}$ then finding the y-coordinate by evaluating $f\left(\frac{-b}{2a}\right)$.

The exception to this is when the quadratic equation is already in the form

$$f(x) = x(x-h)^2 + k$$

In this case, the vertex is (h, k).

Maximum or Minumum of a Quadratic $f(x) = ax^2 + bx + c$

If the graph of a quadratic equation is cup up (a>0), there is a minimum at the vertex. If the graph of a quadratic is cup down (a<0), there is a maximum at the vertex.

Example Find the zeros, y-intercept, shape, vertex, and minumum or maximum of the graph of the quadratic function $f(x) = x^2 - x - 6$. Then sketch the graph of the quadratic.

Zeros: It factors, so

$$\begin{aligned} 0 &= x^2 - x - 6 \\ &= (x-3)(x+2) \end{aligned}$$

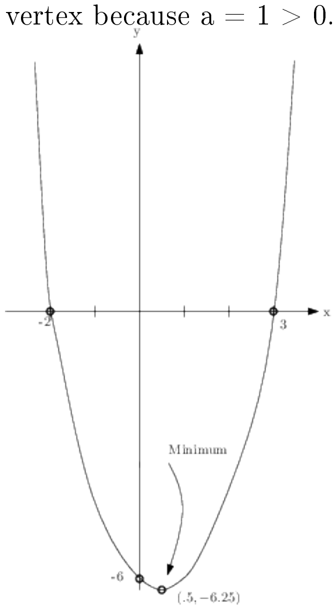
$x - 3 = 0$	$x + 2 = 0$
$x = 3$	$x = -2$

Y-axis intercept:

$$f(0) = 0^2 - x - 6 = -6$$

So, the y-intercept is $(0, -6)$.
Shape: a = 1, so a>0, so it is cup up.
Vertex: $\frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$, and $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6 = \frac{1}{4} - \frac{1}{2} - 6 = \frac{1-2-24}{4} = \frac{-25}{4} = -6.25$
Thus, the vertex is $(.5, -6.25)$

Minumum: There is a minumum at the vertex because a = 1 > 0.



Exercises

Consider the parabola
$f(x) = x^2 - 4x - 5$.

1. Find the x-axis intercepts. (Set f(x) = 0 then solve for x.)
2. Find the vertex. $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
3. Find the axis of symmetry. (x = -b/(2a)).
4. Sketch the graph of this function.
5. Find the minimum (at the vertex.)

Consider the parabola $y = -16x^2 - 64x$.

6. Find the x-axis intercepts. (Set f(x) = 0 then solve for x.)
7. Find the vertex. $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
8. Find the axis of symmetry. (x = -b/(2a)).
9. Sketch the graph of this function.
10. Find the maximum (at the vertex.)