0.1 Quadratic Applications

The trick with almost all word problems is translating the problem (verbage) into a mathematical expression which can be solved by known techniques. In the case of this section, we are focusing on solving quadratic equations and inequalities which arise from application problems.

The general steps are these.

1. Identify the question in the problem.
2. Translate the problem using known values into a quadratic equation, or inequality, with 1 unknown.
3. Solve the equation or inequality.
4. Answer the original question with a practical answer. For example, it would not be wise to try ordering $\sqrt{83}$ tons of steel or even 9.1104336 tons of steel, but ordering 9.2 tons of steel or 10 tons of steel would be well understood.

Example A hotel has 200 rooms each renting for \$100 per night for a total revenue of \$20,000 per night. For each \$1 increase in rent, there is one less guest. For what price range of nightly rent would total revenue be at least \$20,000 ?
We let x be the increase of price over \$100, so the price of a room is (x+100). The number of rooms rented is then (200 - x).
Revenue = (unit price)(units rented) = (x+100)(200-x).
We want revenue at least \$20,000, so we require a solution to the equation

$$(x+100)(200-x) \geq 20000$$

We expand and simplify this quadratic inequality with zero on the left hand side.

$$\begin{aligned} -x^2+100x+20000 &\geq 20000 \\ -x^2+100x &\geq 0 \end{aligned}$$

Solve the equation $x^2 + 100x = 0$ to find the endpoints of our solution to the inequality.

$$\begin{aligned} -x^2+100x &= 0 \\ x(-x+100) &= 0 \end{aligned}$$

$$x = 0 \qquad -x+100 = 0$$

$$x = 100$$

We split the real number line at x = 0 and x = 100 into three intervals, then test a point in each interval:

Interval	x	$(x+100)(200-x)$ ≥ 20000
$(-\infty, 0)$	-1	can't do, hotel full
(0,100)	1	(101)(199)=20099, yes
$(100, \infty)$	101	(201)(99)=19899, no

Thus, the interval (0,100) works, and we already know that x = 0 and x = 100 work, so out solution is [0,100].
To answer the original question, x is the increase in price over \$100, so the range of prices we want is \$100 to \$200 rent per night.

Exercises

1. A hotel charges \$60 per night and is full with 150 guests for a total revenue of \$9000. For each \$1 increase in price, two fewer rooms are rented. For what range of prices will the total revenue be at least \$9000 ?
2. A life saving drug is sold at \$100 to 2000 patients for a total revenue of \$200,000. For each \$1 increase in drug price, 10 fewer patients can afford the drug. What range of prices will keep revenue at least \$200,000 ?
3. A bullet is straight shot up in the air at 1700 ft/sec, and the height of the bullet above ground is given by $y = -16t^2 + 1700t$. When does the bullet hit the ground? Assume there is no air resistance.
4. A golf ball is thrown straight up on the surface of the moon at 5 meters per second. the height of the ball above the surface of the moon is $y = -1.6t^2+5t$. When does the ball return to the surface of the moon?