0.1 Quadratic Inequalities

We can solve the equation $x^2 + x - 6 = 0$ by factoring or another method.

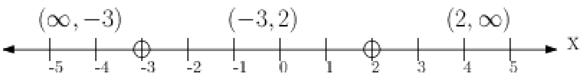
$$\begin{array}{rcl} x^2 + x - 6 & = & 0 \\ (x+3)(x-2) & = & 0 \end{array}$$

$$\begin{array}{c|c} x+3=0 & x-2=0 \\ x=-3 & x=2 \end{array}$$

By what about $x^2+x-6 < 0$ or $x^2+x-6 \geq 0$? In either case, the result will be in the form of intervals or, perhaps, there will be no real solution.

The trick in any case is to find the solution to the equation with zero on one side first, as was done for $x^2 + x - 6 = 0$. This done, we can examine the inequalities $x^2 + x - 6 < 0$, $x^2+x-6 \leq 0$, $x^2+x-6 > 0$, or $x^2+x-6 \geq 0$.

The solutions of $x^2 + x - 6 = 0$, -3 and 2, divide the real number line into three intervals.



The solution for $x^2 + x - 6 < 0$ consists of those intervals which satisfy the inequality. It happens that, for each interval, every point in the intervals works or every point in the intervals fails to satisfy the inequality. Thus, we test one point from each interval to see if the interval is part of the solution. It helps to pick easy points.

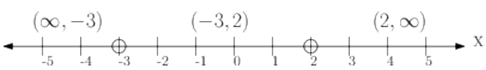
Example Solve the inequality $x^2 + x - 6 < 0$.

We first solve the equation $x^2+x-6 = 0$.

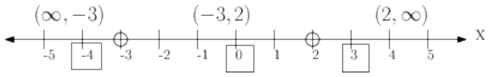
$$\begin{array}{rcl} x^2 + x - 6 & = & 0 \\ (x+3)(x-2) & = & 0 \end{array}$$

$$\begin{array}{c|c} x+3=0 & x-2=0 \\ x=-3 & x=2 \end{array}$$

We then plot the two solutions on the number line and write down the possible interval soutions.



We pick easy points, one per each interval to test.



Test these points -4, 0, 3 in the inequality $x^2 + x - 6 < 0$.

x	$x^2 + x - 6$	
-4	$(-4)^2 - 4 - 6 = 6 \not< 0$	FAIL
0	$0^2 + 0 - 6 = -6 < 0$	Success
3	$3^2 + 3 - 6 = 6 \not< 0$	FAIL

Thus, our solution is the interval $(-3, 2)$.

Exercises

Find the solution for each inequality.

1. $x^2 - x - 6 > 0$
2. $x^2 - 4 \leq 0$
3. $x^2 - 9 \geq 0$
4. $x^2 - 3x < 0$