0.1 Equations Quadratic in Form

Consider the equation $x^4 - x^2 - 6 = 0$. It is a fourth degree equation, yet note that $\left(x^2\right)^2 = x^4$, and if we looked at x^2 and variable itself, this equation is similar to $x^2 - x - 6 = 0$. This can be done formally by substitution.

Example Solve $x^4 - x^2 - 6 = 0$.
Let $u = x^2$, then

$$\begin{aligned} x^4 - x^2 - 6 &= 0 \\ u^2 - u - 6 &= 0 \\ (u-3)(u+2) &= 0 \end{aligned}$$

$u - 3 = 0$	$u + 2 = 0$
$u = 3$	$u = -2$
$x^2 = 3$	$x^2 = -2$
$x = \pm\sqrt{3}$	$x = \pm\sqrt{-2}$
	$x = \pm i\sqrt{2}$

Thus, our solution set is $\left\{\sqrt{3}, -\sqrt{3}, i\sqrt{2}, -i\sqrt{2}\right\}$.

Exercises

Solve the equations by substitution.

1. $x^4 + 7x^2 + 6 = 0$
2. $x^4 - 16 = 0$
3. $81x^4 - 16 = 0$
4. $x^4 - 1 = 0$