0.1 Quadratic Formula

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula solves all quadratic equations, and we obtain it by completing the square. Start with the general quadratic equation.

$$\begin{aligned}
ax^2 + bx + c &= 0 \\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\
-\frac{c}{a} & \quad -\frac{c}{a} \\
x^2 + \frac{b}{a}x &= -\frac{c}{a} \\
\frac{b^2}{4a^2} & \quad \frac{b^2}{4a^2} \\
x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= \frac{b^2}{4a^2} - \frac{c}{a} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
\sqrt{\left(x + \frac{b}{2a}\right)^2} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\
x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Tips on Using the Quadratic Formula

1. It sometimes helps to write down the coefficients a, b, and c.
2. Be sure to include the negative sign with the coefficient as in the following example where c = -9.
3. It is often helpful to use parenthesis around each coefficient a, b, and c.
4. As in the following example, carefully identify common factors for the entire numerator for cancellation.
5. It is usual to leave answers in exact form as in $\frac{-2 \pm \sqrt{31}}{3}$ and $\frac{3 \pm i\sqrt{19}}{4}$, and these are understood to mean the solution sets $\left\{\frac{-2 + \sqrt{31}}{3}, \frac{-2 - \sqrt{31}}{3}\right\}$ and $\left\{\frac{3}{4} + \frac{\sqrt{19}}{4}i, \frac{3}{4} - \frac{\sqrt{19}}{4}i\right\}$ respectively.

Example Solve the equation $3x^2 + 4x - 9 = 0$.
a = 3, b = 4, and c = -9.

$$\begin{aligned}
x &= \frac{-(4) \pm \sqrt{(-4)^2 - 4(3)(-9)}}{2(3)} \\
&= \frac{-4 \pm \sqrt{16 + 108}}{6} \\
&= \frac{-4 \pm \sqrt{124}}{6} \\
&= \frac{-4 \pm \sqrt{4 \cdot 31}}{6} = \frac{-4 \pm \sqrt{4}\sqrt{31}}{6} \\
&= \frac{-4 \pm 2\sqrt{31}}{6} = \frac{2(-2 \pm \sqrt{31})}{2 \cdot 3} \\
&= \frac{-2 \pm \sqrt{31}}{3}
\end{aligned}$$

Example Solve the equation $4x^2 - 6x + 7$.

$$\begin{aligned}
x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)} \\
&= \frac{6 \pm \sqrt{36 - 112}}{8} = \frac{6 \pm \sqrt{-76}}{8} \\
&= \frac{6 \pm 2i\sqrt{19}}{8} = \frac{3 \pm i\sqrt{19}}{4}
\end{aligned}$$

Example Solve the equation $4.2x^2 + 17.1x - 12.5 = 0$

$$\begin{aligned}
x &= \frac{-(17.1) \pm \sqrt{(17.1)^2 - 4(4.2)(-12.5)}}{2(4.2)} \\
&= \frac{-17.1 \pm \sqrt{502.41}}{8.4} \\
&\approx \frac{-17.1 \pm 22.41}{8.4} \\
&\approx -4.70, .632
\end{aligned}$$

Exercises

Solve the following using the quadratic formula.

1. $3x^2 + 4x + 5 = 0$
2. $x^2 + 2x + 4 = 0$
3. $7x^2 - 2x - 10 = 0$
4. $4.21x^2 + 7.82x - 19.3 = 0$
5. $3.5x^2 - 4.1x + 2.8 = 0$
6. $205x^2 + 1020x - 400 = 0$

7. The position s of a bullet shot straight up from the top of a 200 foot cliff above the surface of the sea at 1150 ft/sec is $s = -16t^2 + 1150t + 200$ where t is the time in seconds after the bullet is fired. When does the bullet hit the sea next to the cliff?
   HINT: Set s = 0 on the left side of the equation, then solve for t.