1 Quadratic Equations

1.1 Solving by Factoring or Taking Roots

The equation $x^2 - x - 6 = 0$ can be solved by factoring because, well, it can be factored!

$$\begin{array}{rcl} x^2 - x - 6 & = & 0 \\ (x-3)(x+2) & = & 0 \end{array}$$

$x-3=0$	$x+2=0$
$x=3$	$x=-2$

So the solution set is $\{3, -2\}$.

This sounds silly to say "it can be solved by factoring if it can be factored", but it means that equations that are obviously factorable are easily solved using the zero product rule. Not all factoring problems are obvious, so there are other methods to use in the next few sections. Here, when you are asked to solve by factoring, it should be easily factored.

For a nice rule about factoring, $ax^2 + bx = 0$ is always easy to solve by factoring because x can be factored out

$$\begin{array}{rcl} ax^2 + bx & = & 0 \\ x(ax+b) & = & 0 \end{array}$$

$x=0$	$ax+b=0$
	$ax=-b$
	$x=\dfrac{-b}{a}$

Taking Roots

Taking roots is easy for forms like

$$ax^2 + c = 0$$

These are solved by isolating the x^2 term on one side then taking the square root of both sides in this fashion:

$$\begin{array}{rcl} ax^2 + c & = & 0 \\ -c & & -c \end{array}$$

$$\begin{array}{rcl} ax^2 & = & -c \\ x^2 & = & \dfrac{-c}{a} \\ x & = & \pm\sqrt{\dfrac{-c}{a}} \end{array}$$

Note that we are talking about possibly two solutions for x when we use the $\pm$ symbol: $\sqrt{\dfrac{-c}{a}}$ and $-\sqrt{\dfrac{-c}{a}}$

Example Solve $2x^2 - 10 = 0$

$$\begin{array}{rcl} 2x^2 - 10 & = & 0 \\ 2x^2 & = & 10 \\ x^2 & = & 5 \\ x & = & \pm\sqrt{5} \end{array}$$

Note that $-\sqrt{5}$ and $\sqrt{5}$ are both solutions because $2\left(-\sqrt{5}\right)^2 - 10 = 10 - 10 = 0$, and $2\left(\sqrt{5}\right)^2 - 10 = 10 - 10 = 0$.

Example Solve $3x^2 + 2 = 0$

$$\begin{array}{rcl} 3x^2 + 2 & = & 0 \\ 3x^2 & = & -2 \\ x^2 & = & \dfrac{-2}{3} \\ x & = & \pm\sqrt{\dfrac{-2}{3}} = \pm i\sqrt{\dfrac{2}{3}} \end{array}$$

Exercises

Solve the following using the zero product rule.

1. $(x-2)(x+7) = 0$
2. $x^2 + x - 12 = 0$
3. $x^2 - x - 6 = 0$
4. $5x^2 - 6x + 1 = 0$
5. $6x^2 - 12x = 0$
6. $2x^2 + 7x = 0$
7. $x^2 = 9x - 20$

Solve the following by taking roots.

8. $x^2 = 64$
9. $4x^2 - 20 = 0$
10. $2x^2 - 8 = 0$
11. $2x^2 - 3 = 0$
12. $x^2 + 5 = 0$
13. $7x^2 + 5 = 0$
14. $(2x - 5)^2 = 49$