0.1 Radical Equations

Almost the entire trick with radical equations is raising both sides of the equation to the same power. It sometimes is this easy.

Example Solve the equation for x: $\sqrt{x+1} = 3$.

$$\begin{aligned} \sqrt{x+1} &= 3 \\ \left(\sqrt{x+1}\right)^2 &= 3^2 \\ x+1 &= 9 \\ x &= 8 \end{aligned}$$

On checking in the original equation, we have success: $\sqrt{8+1} = \sqrt{9} = 3 \surd$.

However, it is important to check the result from such a step (rasing both sides on an equation to a power) to ensure that it really works in the equation. It does not always work, and sometimes it is not obvious why.

Sometimes it is obvious. Consider the equation $\sqrt{x+2} = -5$. There is a problem with this to begin with, and the problem is that the square root radical means the principal square root which is always positive, so we cannot obtain -5 as a result for any square root, Nevertheless, let us apply the technique of square both sides of this equation and see.

$$\begin{aligned} \sqrt{x+2} &= -5 \\ \left(\sqrt{x+2}\right)^2 &= (-5)^2 \\ x+2 &= 25 \\ x &= 23 \end{aligned}$$

On checking, $\sqrt{23+2} = \sqrt{25} = 5 \neq -5$. Thus, our solution set is empty or Ø. 23 was an extraneous solution.

Exercises

1. Solve the following radical equations. $\sqrt{x} = 7$
2. $\sqrt{x-4} = 9$
3. $\sqrt[3]{2x+1} = -3$
4. $\sqrt{x-4} - 7 = 0$
5. $\sqrt{x+3} + \sqrt{x-1} = 2$
6. The relationship between the length L of a pendulum in meters and the period T in seconds is $T = 2\pi\sqrt{\frac{L}{9.8}}$.

   If the period is 7.3 seconds, what is the length of the pendulum?
7. Solve for x: $\sqrt{4x+1} = 5$
8. How long does a pendulum have to be in meters on the surface of the Earth to have a period of 2.2 seconds ? $T = 2\pi\sqrt{\frac{L}{9.8}}$