0.1 Variation

Regarding people who sell things and earn commission, we say that their pay varys directly with sales. Suppose p is pay in dollars, and s is sales in dollars, then we can write this relationship between p and s as

$$p = ks$$

where k is a constant of proportionality. For people who sell things on a commission, k is called the commission rate. Once k is set (the commission rate), pay p depends entirely on sales s.

Example A person who sells houses earns a 3% commission on sales. What does this house seller get paid for selling a $145,000 house?
We have k for this problem, and written as a decimal 3% = .03. Our equation of variation is

$$p = .03s$$

For $s = 150000$,we have

$$p = .03(150000) = 4500$$

Thus, the sales commission was $4500.

Example A telephone seller earns a 15% commission for every cell phone contract sold. If the seller got paid $2460 for commission one month, what was the sales to earn this commission?
We have k = 15% = .15, and our equation of variation is

$$p = .15s$$

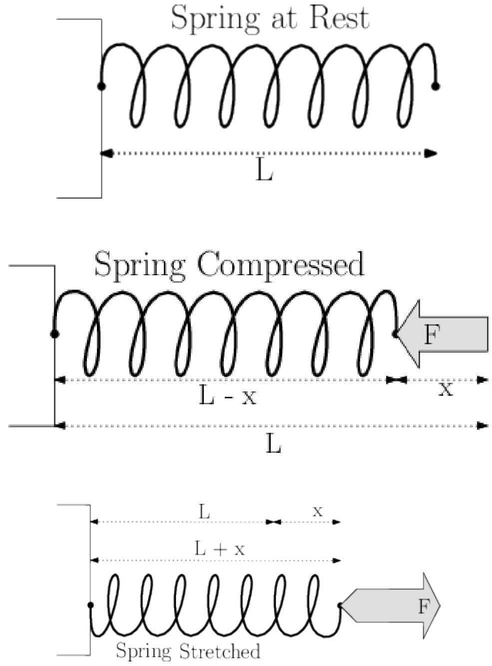
Here we know that p = 2460, and we need to find s.

$$\begin{aligned} 2460 &= .15s \\ \frac{2460}{.15} &= s \\ s &= \end{aligned}$$

We call k in this kind of variation problem the constant of proportionality

Spring - Hooke's Law

An ordinary spring has a natural length L when there is no compressing or stretching force applied. Surprisingly, springs require the same force for compression and stretching to change the length of the spring the same amount x.



The relationship between the change x of the length of a spring from compression or stretching caused by a force F is

$$x = kF$$

where the constant of proportionality depends on the spring itself. This is called Hooke's Law.

Example A truck coil spring is 14 inches long at rest. It is compressed to 12 inches in length with a force of 300 lb. What will be the length of the spring with a force of 1000 lb?
We need to find the constant k first using Hooke's law. Note that the natural length is 14, so x is 2 in when the spring is compressed to 12 in.

$$\begin{aligned} x &= kF \\ 2 &= k(300) \\ k &= \frac{2}{300} \approx .00667 \end{aligned}$$

The constant k is $0\dot{0}667\frac{in}{lb}$. This constant along with its unit ration $\frac{in}{lb}$ says that the spring compresses .00667 in for every lb of

force.
How far will the spring compress with a force of 1000lb?

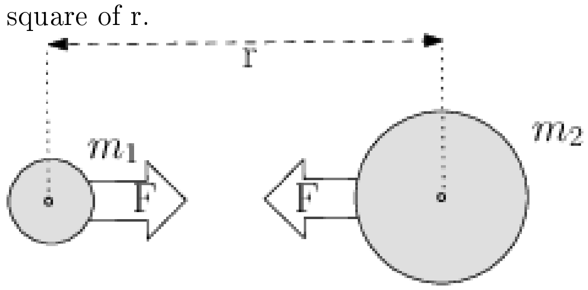
$$x = .00667(1000) = 6.67in$$

Thus, the length of the spring is reduced by 6.67in: 14in - 6.67in = 7.33in. the spring is 7.33in long when compressed with a force of 1000lb.

Inverse Variation

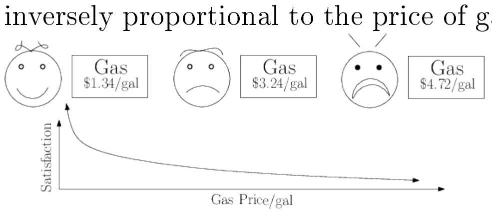
The force of gravity between two objects decreases with the square of the distance between them (Newton's law of gravity). The electrostatic force between two charged particles decreases with the square of the distance between them (Coulomb's law). The volume of a gas is directly proportional to the gas temperature, directly proportional to the number of gas molocules, and inversely proportional to the gas pressure.

Example The Force of Gravity between masses $m_1$and m_2 separated by a distance r is directly proportional to $m_1$and m_2, and inversely proportional to the square of r.



$$F = G\frac{m_1 m_2}{r^2}$$

Example Gas station customer satisfaction is inversely proportional to the price of gas.

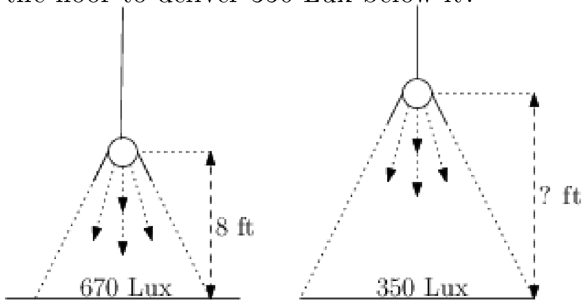


These physical and psychological relationships can be expressed in precise ways, yet each has a constant of proportionality which must be determined for each relationship. Once the constant of proportionality is determined, any quantity in the relationship can be found if the others are known.

Example The Lux L (a measure of light per unit of area) of a light source varies inversely with the square of the distance d of the light source from the object of illumination where the Lux is measured.

$$L = k\frac{1}{d^2}$$

A light bulb suspended 8 feet over the floor of a high garage and delivers 670 Lux to a spot on the floor directly below the light bulb. 350 Lux would be enough for the floor directly below the bulb, and raising the bulb higher would increase the spread of illumination in the garage. How high can the bulb be raised above the floor to deliver 350 Lux below it?



We must find k first.

$$\begin{aligned} 670 &= k\frac{1}{8^2} \\ k &= 670 \cdot 8^2 = 42880 \approx 43,000 \end{aligned}$$

The units of k are $\frac{x}{ft^2}$, and $k = 43,000\frac{Lux}{ft^2}$, and our relationship between L and d is

$$L = 43000 \cdot \frac{1}{d^2}$$

We want to find d so that L = 350.

$$\begin{aligned} 350 &= 43000 \cdot \frac{1}{d^2} \\ d^2 &= \frac{43000}{350} \approx 122.86 \\ d &\approx \sqrt{122.86} \approx 11.085 \end{aligned}$$

Rounding to two significant digits, we find that d = 11ft to produce a Lux of 350 on the floor below the light bulb.

Exercises

The length x of a spring's stretch with a weight W attached is proportional to the weight W.

$$x = kW$$

1. A particular spring stretches 3 cm with a 4 lb weight attached.
   Find k
2. Write the relationship between the length x and the weight W using the value of k you obtained.
3. How far will the spring stretch with a weight of 13 lb?
4. If the spring stretches 7.15 cm, what is the value of W?
5. If the spring is made stiffer, the value of k will be different. Would k be larger or smaller for a stiffer spring?

Newton's law of gravitation expresses the relationship between two masses m_1 and m_2, the distance r between the masses, and the resulting gravitional force F between the masses.

$$F = G\frac{m_1 m_2}{r^2}$$

G is the universal gravitational constant.

6. If $m_1 = 5.97 \times 10^{24} kg$, $m_2 = 1.99 \times 10^{30} kg$, $r = 1.496 \times 10^{11} m$, and $F = 3.54 \times 10^{22}\ N$ (N is the unit of force Newton), find G.
7. For $m_1 = 7 \times 10^{22} kg$, $m_2 = 5.97 \times 10^{24}$, and $r = 3.84 \times 10^8 m$, find the gravitational force F between these two masses using the value of G obtained above.