0.1 Rational Equations

Rational equations are actually easier to solve than it is to simplify and perform arithmetic with rational expressions. The entire trick is to clear fractions at the very first on both sides of the equation by multiplying both sides by the LCD of all denominators.

However, because rational expressions have denominators, we must ensure that the results of solving equations work in the original equations, and this usually means that we cannot allow division by zero. Results from solving rational equations that cause division by zero are called extraneous solutions to the equation. Extraneous solutions arise from correct work, all steps in solving the equations carried out just fine, but extraneous solutions do not work. Extraneous solutions here cause division by zero and are not offered as answers to rational equations.

Example Solve the equation $\frac{3}{x+2} = \frac{x+1}{2x}$

The LCD is $(x+2)2x$, so our first step will be to multiply both sides of the equation by $(x+2)2x$ to clear denominators, then we solve as usual for an equation.

$$\begin{aligned}
\frac{3}{x+2} &= \frac{x+1}{2x} \\
\frac{3}{x+2}\cdot(x+2)2x &= \frac{x+1}{2x}\cdot(x+2)2x \\
\frac{3(x+2)2x}{(x+2)} &= \frac{(x+1)(x+2)2x}{2x} \\
\frac{3\cancel{(x+2)}2x}{\cancel{(x+2)}} &= \frac{(x+1)(x+2)\cancel{2x}}{\cancel{2x}} \\
3\cdot 2x &= (x+1)(x+2) \\
6x &= x^2+3x+2 \\
-6x &\quad -6x \\
0 &= x^2-3x+2 \\
0 &= (x-2)(x-1)
\end{aligned}$$

We will solve this equation $0 = (x-2)(x-1)$ using the zero product rule.

$$\begin{array}{cc} x-2=0 & x-1=0 \\ x=2 & x=1 \end{array}$$

Thus, our resulting solution set is $\{2,1\}$. Now, check these in the original.

$$\begin{aligned}
x=2: \quad \frac{3}{x+2} &= \frac{x+1}{2x} \\
\frac{3}{2+2} &\stackrel{?}{=} \frac{2+1}{2\cdot 2} \\
\frac{3}{4} &= \frac{3}{4} \qquad \surd
\end{aligned}$$

$$\begin{aligned}
x=1: \quad \frac{3}{x+2} &= \frac{x+1}{2x} \\
\frac{3}{1+2} &\stackrel{?}{=} \frac{1+1}{2\cdot 1} \\
\frac{3}{3} &= \frac{2}{2} \qquad \surd
\end{aligned}$$

Exercises

Solve the following equations for x.

1. $\frac{2}{x} = \frac{4}{x+1}$
2. $\frac{4}{5x-1} = \frac{2}{2x-1}$
3. $\frac{5}{3x-8} = \frac{x}{x+2}$
4. $\frac{x}{2} = \frac{2x+1}{3}$
5. $\frac{6}{x} - 2 = 1$

Solve each problem using algebra.

6. Bob can mow a lawn in 4 hours. If Bob mows the lawn with Sally, they mow the lawn in 3 hours. How long would it take Sally to mow the lawn by herself? (Use algebra for credit.)
7. A biker travels 20 miles in the same amount of time it takes a hiker to walk 6 miles. If the rate of the biker is 7 mph faster than the hiker, what is the rate of the biker? (You must use algebra for credit.)