0.1 Complex Fractions

We are talking about fractions of fractions here, things like $\frac{\frac{3}{x}-\frac{1}{x^2}}{\frac{2}{x^2}+4}$. There are several ways to deal with these things, but we will look at one fairly easy way to simplify these items and turn them into simple fractions of polynomials. The technique here is to find the LCD of the denominators of all the smaller fractions, then multiply the big numerator and denominator by this LCD in order to clear denominators of all the smaller fractions.

Example Simplify $\frac{\frac{3}{x}-\frac{1}{x^2}}{\frac{2}{x^2}+4}$.

The LCD of all the smaller denominators (x, x^2, x^2) is x^2, so we start off by multiplying the big numerator and denominator both by this LCD of x^2. This is the same as multiplying the entire fraction by a form of 1.

$$\begin{aligned}
&= \frac{\frac{3}{x}-\frac{1}{x^2}}{\frac{2}{x^2}+4} \\
&= \frac{\left(\frac{3}{x}-\frac{1}{x^2}\right)x^2}{\left(\frac{2}{x^2}+4\right)x^2} \\
&= \frac{\frac{3\cdot x^2}{x}-\frac{1\cdot x^2}{x^2}}{\frac{2\cdot x^2}{x^2}+4\cdot x^2} \\
&= \frac{\frac{3\cdot x^{1\cancel{2}}}{\cancel{x}}-\frac{1\cdot \cancel{x^2}}{\cancel{x^2}}}{\frac{2\cdot \cancel{x^2}}{\cancel{x^2}}+4\cdot x^2} \\
&= \frac{3x-1}{2+4x^2} \\
&= \frac{3x-1}{2(2x^2+1)}
\end{aligned}$$

Note that we factored the final answer to see if there were any common factors between the numerator and denominator. There were not any common factors, and so we have the simplified result in the end.

Exercises

Simplify.

1. $\frac{1+\frac{2}{x}}{2-\frac{1}{x}}$

2. $\frac{2+\frac{1}{x-3}}{3-\frac{5}{x-3}}$

3. $\frac{1-\frac{5}{x}+\frac{6}{x^2}}{1-\frac{9}{x^2}}$

4. For fun, simplify: $1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$