0.1 Addition and Subtraction of Rational Expressions

Adding rational expressions is also the same as adding fractions of integers. the key is making the denominators to be exactly the same, then the addition or subtraction is carried out between numerators over the common denominator.

$$\begin{aligned} \frac{a}{c}+\frac{b}{c} &= \frac{a+b}{c} \\ \frac{a}{c}-\frac{b}{c} &= \frac{a-b}{c} \end{aligned}$$

Example Add then simplify: $\frac{3}{x^2+1}+\frac{x+8}{x^2+1}$

$$\begin{aligned} \frac{3}{x^2+1}+\frac{x+8}{x^2+1} &= \frac{3+x+8}{x^2+1} \\ &= \frac{x+11}{x^2+1} \end{aligned}$$

Example Perform the operations then simplify: $\frac{3x}{x-1}+\frac{4}{x-1}-\frac{x+6}{x-1}$

$$\begin{aligned} &= \frac{3x}{x-1}+\frac{4}{x-1}-\frac{x+6}{x-1} \\ &= \frac{3x+4-(x+6)}{x-1} \\ &= \frac{3x+4-x-6}{x-1} \\ &= \frac{2x-2}{x-1} \\ &= \frac{2(x-1)}{(x-1)} = \frac{2\cancel{(x-1)}}{\cancel{(x-1)}} \\ &= \frac{2}{1} = 2 \end{aligned}$$

Note that the addition and subtraction was performed first with the common denominators. Then, the numerators was simplified, and the distributive law was applied carefully so that $-(x+6)$ became $-x-6$. Then, the numerator and denominator were factored and common factors cancelled out to obtain the final, simplified result.

Example Perform the operations then simplify: $\frac{2x}{x-1}-\frac{3}{2x+3}$

The LCD of the denominators is $(x-1)(2x+3)$, so we multiply each fraction by a form of 1 to make the denominators common. The first fraction will be multiplied by $\frac{2x+3}{2x+3}$, and the second fraction will be multiplied by $\frac{x-1}{x-1}$; this is done so both denominators will end up being the same, namely $(x-1)(2x+3)$.

$$\begin{aligned} &= \frac{2x}{x-1}-\frac{3}{2x+3} \\ &= \frac{2x}{x-1}\cdot\frac{2x+3}{2x+3}-\frac{3}{2x+3}\cdot\frac{x-1}{x-1} \\ &= \frac{2x(2x+3)}{(x-1)(2x+3)}-\frac{3(x-1)}{(2x+3)(x-1)} \end{aligned}$$

Here, the fractions have the same denominator, the the operation os completed between the numerators over the common denominator.

$$\begin{aligned} &= \frac{2x(2x+3)}{(x-1)(2x+3)}-\frac{3(x-1)}{(2x+3)(x-1)} \\ &= \frac{2x(2x+3)-3(x-1)}{(x-1)(2x+3)} \\ &= \frac{4x^2+6x-3x+1}{(x-1)(2x+3)} \\ &= \frac{4x^2+3x+1}{(x-1)(2x+3)} \end{aligned}$$

The numerator $4x^2+3x+1$ does not factor further, so the simplified result is $\frac{4x^2+3x+1}{(x-1)(2x+3)}$. It is usual to leave answers factored in this form.

Exercises

Perform the operations then simplify.

1. $\frac{2}{3}-\frac{5}{4}$
2. $\frac{3x+1}{x^2-4}+\frac{x^2-x}{x^2-4}$
3. $\frac{2x-3}{2x}+\frac{x+3}{3x}$
4. $\frac{4x}{2x-1}-\frac{5}{x-6}$
5. $\frac{y}{x-y}+2-\frac{x}{y-x}$
6. $\frac{3x+5}{x+5}+\frac{x+1}{x-2}-\frac{4x^2-3x-1}{x^2+3x-10}$

7. $\dfrac{2x}{x^2+3x-10} - \dfrac{4}{x^2+3x-10}$

8. $\dfrac{3}{x} + \dfrac{x+1}{x-4}$