0.1 Zero Product Rule

Theorem If $A \cdot B = 0$, then $A = 0$ and $B = 0$ are both solutions.

This means that if you can factor an expression on one side of an equation with zero on the other side, you can solve possibly simpler equations. In the case of polynomials which factor, the zero product rule allows you to solve equations with lower degree. In our problems here, the goal is to end up solving linear equations.

Example Solve $(x-3)(x+7) = 0$

This is already factored, so use the zero product rule to set each factro equal to zero, then solve each equation.

$$\begin{array}{c|c} x-3=0 & x+7=0 \\ x=3 & x=-7 \end{array}$$

Thus, the soution set if $\{3, -7\}$.

Example Solve $x^2 - 4x - 5 = 0$

This has to be factored first.

$$\begin{aligned} x^2 - 4x - 5 &= 0 \\ (x-5)(x+1) &= 0 \end{aligned}$$

Now, set each factor equal to zero and solve.

$$\begin{array}{c|c} x-5=0 & x+1=0 \\ x=5 & x=-1 \end{array}$$

Thus, the solution set is $\{5, -1\}$.

Example Solve $4x^2 + 1 = 5x$.

There is no point in trying to use the zero product rule unless there is a zero by itself on one side, so we bring everything to one side first, then factor.

$$\begin{aligned} 4x^2 + 1 &= 5x \\ 4x^2 - 5x + 1 &= 0 \\ (4x-1)(x-1) &= 0 \end{aligned}$$

Now, set each factor equal to zero and solve.

$$\begin{array}{c|c} 4x-1=0 & x-1=0 \\ 4x=1 & x=1 \\ x=\dfrac{1}{4} & \end{array}$$

The solution set is $\left\{\frac{1}{4}, 1\right\}$.

Exercises

Solve the following problems by factoring. You must have zero on one side in order for the zero-product rule to work.

1. $x(x-6) = 0$
2. $x^2 - x - 6 = 0$
3. $x^2 - 49 = 0$
4. $3x - 7x^2 = 0$
5. $x^2 + 4x = 5$
6. $x(x+3) = 28$
7. The distance s an object has fallen in t seconds after being thrown down with velocity v is $s = vt + 16t^2$.

   If a bowling ball is thrown straight down at 32 ft per second from a 1920 foot high cliff. In how many seconds will the bowling ball land on a tin can at the bottom of the cliff? Assume no air resistance.

Find the solution set for the following equations.

8. $x^2 + x - 2 = 0$
9. $x^2 + 6x + 5 = 0$
10. $x^2 - 5x = 0$
11. $x^2 = 9$
12. $x(2x+1) = 0$
13. $x^2 - 3x - 4 = 0$
14. $x^2 - 4x - 21 = 0$
15. $x^2 - 4x = 0$
16. $6x^3 - 12x^2 = 0$
17. $(2x-7)(x^2-16) = 0$
18. $15y^2 + y - 2 = 0$
19. $t^2 - t - 12 = 0$
20. $z^2 - 8z + 12 = 0$
21. $k^2 + 7k + 12 = 0$
22. $6c^2 - 17c + 12 = 0$
23. $3x^2 + 5x^5 = 0$
24. $7x^2 - 20x = 0$

25. $\mathrm{x}(\mathrm{x}+2)+7(\mathrm{x}+2)=0$
26. $x^2+2x+2x+4=0$
27. $(\mathrm{x}-2)(\mathrm{x}+5)=0$
28. $(\mathrm{x}-5)(\mathrm{x}+5)=0$
29. $(\mathrm{x}+4)(\mathrm{x}+5)(\mathrm{x}+6)=0$
30. $x^2+7x+6=0$
31. $x^2-5x-6=0$
32. $x^2-49=0$
33. $y^2-6y+9=0$
34. $5x^2-5x-30=0$
35. $2x^4-2x^3-12x^2=0$