6.3 Multiplication of Polynomials

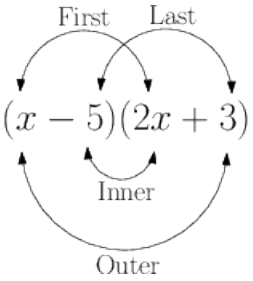
Multiplication of polynomials is a term-by-term process. The product of the terms $2x$ and $3x^2$ is $(2x)(3x^3) = 2 \cdot 3 \cdot x \cdot x^3 = 6x^4$. Multiplying polynomials with more than one term requires repeated application of the distributive law: $A(B+C) = AB+AC$. The distributive law applys to any number of terms, so $A(B + C + D) = AB + AC + AD$ and $A(B-C-D) = AB-AC-AD$ and so forth.

Example Multiply: $2x(3x^2 + 5x - 9)$.
$2x(3x^2 + 5x - 9) = 2x \cdot 3x^2 + 2x \cdot 5x - 2x \cdot 9 = 6x^3 + 10x^2 - 18x$. Notice that we wrote down the third product $-2x \cdot 9$ by observing that a positive times a negative is positive.

Multiplying a polynomial times a polynomial when both have more than one term can also be done using the distributive law.

Example Multiply $(2x - 5)(4x + 7)$.
$(2x - 5)(4x + 7) = (2x - 5)4x - (2x - 5)7 = 2x \cdot 4x - 5 \cdot 4x - [2x \cdot 7 + 5 \cdot 7] = 8x^2 - 20x - 14x - 35 = 8x^2 - 34x - 35$.

Multiplying two binomials , e.g. $(x - 5)(2x + 3)$, is such a common task that there is a special method to it called FOILing for First Outer Inner Last.



$(x-5)(2x+3) = x(2x)+x(3)-5(2x)-5(3) = 2x^2 + 3x - 10x - 15 = 2x^2 - 7x - 15$.

The result is the same as using the distributive law. Another way of looking at any product of two polynomials is to take the product of every term in the first with every term in the second.

For larger polynomial products, say $(2x^2-6x+5)(x^2 - x + 9)$, it is probably easiest to multiply using columns. Column multiplication is essentially identical to ordinary column multiplication of multi-digit numbers except that there are no carrys.

Example Multiply $(2x^2 - 6x + 5)(x^2 - x + 9)$.

		$2x^2$	$-6x$	$+5$
$\otimes$		x^2	$-x$	$+9$
		$18x^2$	$-54x$	45
	$-2x^3$	$6x^2$	$-5x$	
$2x^4$	$-6x^3$	$5x^2$		
$2x^4$	$-8x^3$	$29x^2$	$-59x$	45

Exercises

Multiply out the following, combine like terms, and write in descending order.

1. $3x(x^2 - 4x + 7)$
2. $-2x(3x^2 - 5x - 9)$
3. $(3x - 7)(9x + 5)$
4. $(4x - 3)(4x + 3)$
5. $(2x + 7)^2$
6. $(x - 3y)^2$
7. $(x + 2)^2$

Multiply vertically.

8. $(3x^2 - 4x + 7)(2x - 4)$
9. $(x^2 - 6)(2x^2 + 5x + 9)$
10. $(2x^2 - 3x - 5)(3x^2 + x - 7)$