6 Polynomials

A polynomial is of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ where n is a non-negative integer and the coefficients $a_k$are numbers. If all coefficient are real, then we have a polynomial with real coefficients. Similarly, we can classify polynomials with rational coefficients or integer coeficients.

Each $a_k k^k$is called a term. A term is a product of its coefficient and a variable expression with integer exponents. The degree of a term is the highest power of the variable or, if there is more than one variable, the sum of the exponents of all variables. The term $3x^4$ has degree 4, $5x^2y^3$has degree 5, $10x$has degree 1, and a constant 13 has degree zero. The degree of a polynomial is the highest degree of any term making up the polynomial.

The polynomial $2x^2 - 6x - 7$ has three terms and is called a trinomial. Note that the coefficients include any negative signs, so its first degree term is $-6x$ having a coefficient of -6. The polynomial $2x + 5$ is a binomial because it has two terms, and $4x^4$ is a monomial because it is made up of one term.

Terms of a polynomial can be written in any order, yet it is usual to write them in descending order, left to right, as in $3x^4 - 2x^2 + 7x - 5$.

6.1 Exponents

Exponents or powers mean repeated multiplication. $a^2 = a \cdot a$, $a^3 = a \cdot a \cdot a$, and so on. In general, $a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}$; a multiplied by itself n times. Further, $a^1 = a$. An exponent of 1 is not usually written, but you have to know that it is there. Also, $a^0 = 1$if a is not equal to zero. Here we are talking about integer exponents.

Five hundred years ago, x^2might have been written as xx, x^3as xxx, and so forth. This notation is correct, but it becomes cumbersome for large integer exponents. (Further, we can extend the rules of exponents to apply to non-integer powers later on.)

There shortcuts available using exponents. One could find the product $xxxxx \cdot xxx$ by simply removing the multiplication dot and get $xxxxxxxx$. In exponent form, the problem is x^5x^3, and we add the exponents to get the result. $x^5x^3 = x^{5+3} = x^8$.

Here are the major exponent rules.

$a^n a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^{n-m}$

$(a^n)^m = a^{n \cdot m}$

$(ab)^n = a^n b^n$

Example. Simplify $\left(2x^3\right)^4$.

$$\left(2x^3\right)^4 = 2^4 \left(x^3\right)^4 = 16x^{3 \cdot 4} = 16x^{12}.$$

Scientific Notation

Very large and very small numbers are inconvenient to express as decimals. By inconvenient, we mean both difficult to grasp by appearance and difficult to use for practical calculations. For example, the mass of The Earth is about 5,973,600,000,000,000,000,000,000 kg. Even rounded to one significant digit, this is a rather awkward number to say (6 million million million million kg, or 6 trillion trillion kg), and a mistake of a single decimal place would make huge errors in results. Much easier is the mass of the Earth in scientific notation as $5.9736 \times 10^{24} kg$ or, rounded to one significant digit, as $6 \times 10^{24} kg$.

Scientific notation makes it easy to compare large and small quantities. For example, the mass of the sun is about $2 \times 10^{30} kg$, so it is clear that the Sun is more massive than The Earth. $\left(2 \times 10^{30}\right) \div \left(6 \times 10^{24}\right) \approx 330,000$, so the Sun is roughly 330,000 times more massive than the Earth. This division is accomplished using the scientific notation features of scientific calculators. Few calculators will even allow you to enter a number like 2,000,000,000,000,000,000,000,000,000,000.

Small numbers, likewise, can be represented in scientific notation by using negative exponents. The mass of a proton is about .000 000

000 000 000 000 000 000 001 672 6 kg. Spacing every 3d digit is done to make counting easier, but scientific notation eliminates the problem: $1.672 \times 10^{-27} kg$.

Definition A number x in scientific notation is expressed as a number A such that $1 \leq A < 10$ times ten to an integer power n.

$$A \times 10^n$$

Example Write 0.000 000 000 51 in scientific notation.
Move the decimal point so that there is one ono-zero digit to the left of the decimal point, then count the number of place the decimal point. The decimal point is moved ten place to the left, so $0.00000000051 = 5.1 \times 10^{-10}$.

Example Write 602,214,150,000,000,000,000,000 as a decimal.
The decimal point is considered to be there at the end if ommitted, as in 602,214,150,000,000,000,000,000. , and we need to move it 23 places to the right, so $602,214,500,000,000,000,000,000. = 6.0221415 \times 10^{23}$.

Exercises

Simplify the following.

1. $x^5 x^7$
2. $\dfrac{x^{12}}{x^{20}}$
3. $\left(x^5\right)^3$
4. $(3x)^2$
5. $x^3 x^5 x^7$
6. $\left(3x^2\right)^3$
7. $\dfrac{3x^2 y^6}{6x^8 y}$
8. $\left(x^2 y^{-5}\right)^{-3}$
9. $\dfrac{(4x^2 y)^2}{(x^7 y^{-3})^3}$
10. $x^{2n} x^{n-3}$
11. $\left(\dfrac{\left(x^{-3}\right)^3}{\left(x^2 y^{-1}\right)^{-2}}\right)^2$

Convert to scientific notation.

12. .000 000 000 000 000 023
13. 21,300,000,000,000.

Convert to a decimal.

14. $3.56x10^{-5}$
15. $1.02x10^6$

Calculate.

16. $\left(2.33x10^{16}\right)\left(5.98x10^{-45}\right)$