5.5 Systems of Linear Inequalities

Systems of linear inequalities, as in

$$\left\{ \begin{array}{rcl} x+y & \leq & 12 \\ x & \geq & y \end{array} \right\}$$

occur frequently in practical problems. Real applications may involve many variables, as many as 50, but we will look at problems involving 2 variables. The methods of solution are similar in principle, and we can draw and more easily see what is going on with inequalities in the plane.

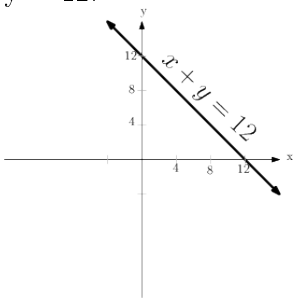
Example Suppose that a canoe shop makes 2 and 3 seat canoes. It can make at most 12 canoes per week, and it must make at least as many 2 seat as 3 seat canoes.
If we let x = number of 2 seat canoes, and y = number of 3 seat canoes, we arrive at the system

$$\left\{ \begin{array}{rcl} x+y & \leq & 12 \\ x & \geq & y \end{array} \right\}$$

In this as in many practical problems, it makes no sense for the variables to be negative, so we usually add on inequalities which restrict the variables to the upper right quadrant of the plane.

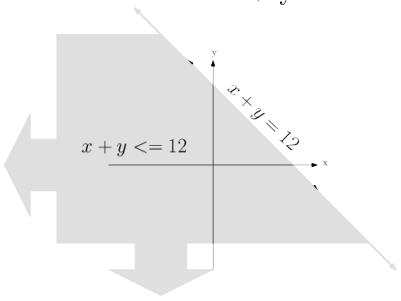
$$\left\{ \begin{array}{rcl} x+y & \leq & 10 \\ x & \geq & y \\ x & \geq & 0 \\ y & \geq & 0 \end{array} \right\}$$

Recall that in graphing an inequality, we end up with a half plane. Here is the graph of x + y = 12:



To graph the half plane $x + y \leq 12$, we test a point which is on one side or the other of this line by plugging the point values of x and y into the inequality. If the point produces a true statement, the plane is on the side of the line with point. If the point produces a false statement, the plane is on the side of the line without the point.

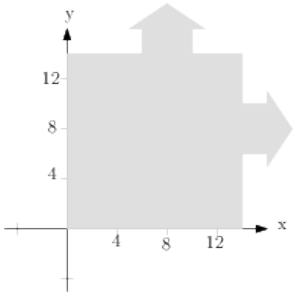
The easiest point, (0,0), is clearly not on the line x + y = 12, so we test it:
$0 + 0 = 0 \leq 12$. True. Thus, the graph of the inequality $x + y \leq 12$ is the half plane containing the point (0,0), and this inequality contains the line x + y = 12.



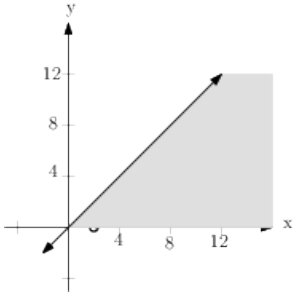
The two inequalities

$$\left\{ \begin{array}{rcl} x & \geq & 0 \\ y & \geq & 0 \end{array} \right\}$$

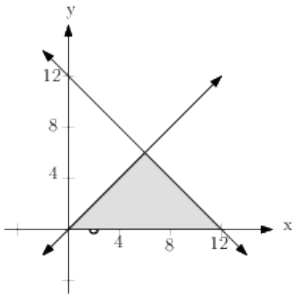
restrict our focus to the upper right quadrant.



In graphing $x \geq y$, we can't test the point (0,0) because it is on the line, but (2,0) will work.
Here, in testing (2,0), we obtain: $2 \geq 0$, true. Thus, we graph the line $x = y$, then we shade the side of the line including the point (2,0).



Combining these three graphs together, where they overlap (where the inequalities are all true), we have



Exercises

Graph the following systems of linear inequalities.

1. $\left\{ \begin{array}{c} x \geq 1 \\ y \geq 2 \end{array} \right\}$
2. $\left\{ \begin{array}{c} x + y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{array} \right\}$
3. $\left\{ \begin{array}{c} 2x + 3y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{array} \right\}$
4. $\left\{ \begin{array}{c} x + y \leqq 100 \\ x \leq 2y \\ x \geq 0 \\ y \geq 0 \end{array} \right\}$

Write the possible values for the variables x and y as a system of linear inequalities.

5. A shop sells coffee and expresso. The total number of cups sold per day is at most 500, and there must be at least as many cups of coffee sold as expressos. Let x = number of cups of coffee sold. Let y = number of expressos sold.
6. A gas station sells regular and super grades of gas. It can pump at most 12,000 gallons per day. To stay viable, it must pump at least 1000 gallons per day. Let x = gallons of regular, and y = gallons of super.