5.2 Solving Systems by Elimination

Consider the linear system

$$\begin{cases} 2x - 3y = 7 \\ x + y = 1 \end{cases}$$

The method of elimination starts out as in substitution by assuming that all equations are true at the same time, so the left side equals the right side for every equation. We want to add some multiple of one equation to the other so that one variable is eliminated, then the resulting equation in one variable is solved. We can find the value of the other variable by repeating elimination, only this time eliminating the other variable.

Example Solve the system $\begin{cases} 2x - 3y = 7 \\ x + y = 1 \end{cases}$ by elimination.

Let us eliminate y. We note that multiplying the second equation by 3 would result in another equation which on addition to the frist equation would eliminate y.

$$\begin{array}{rrcl} & 2x - 3y & = & 7 \\ \text{(multiply by 3)} & x + y & = & 1 \\ \\ & 2x - 3y & = & 7 \\ \oplus & 3x + 3y & = & 3 \\ \hline & 5x & = & 10 \\ & \frac{5x}{5} & = & \frac{10}{5} \\ & x & = & 2 \end{array}$$

Thus, $x = 2$. Now, do it again by eliminating x this time. Note that multiplying the second equation by -2 would allow x to be eliminated.

$$\begin{array}{rrcl} & 2x - 3y & = & 7 \\ \text{(multiply by -2)} & x + y & = & 1 \\ \\ & 2x - 3y & = & 7 \\ \oplus & -2x - 2y & = & -2 \\ \hline & -5y & = & 5 \\ & \frac{-5y}{-5} & = & \frac{5}{-5} \\ & y & = & -1 \end{array}$$

Thus, we have the solution x = 2 and y = -1. This solution of a linear system is usually written as the point $(2, -1)$.

Example Solve the system $\begin{cases} 3x - 4y = 11 \\ 2x + 2y = -3 \end{cases}$ by elimination.

We multiply row one by 2 and row two by -3 so that x is cancelled.

$$\begin{array}{rcl} 6x - 8y & = & 22 \\ -6x - 6y & = & 9 \\ \hline -14y & = & 31 \\ y & = & -\frac{31}{14} \end{array}$$

We multiply row two by 2 so that y is cancelled out.

$$\begin{array}{rcl} 3x - 4y & = & 11 \\ 4x + 4y & = & -6 \\ \hline 7x & = & 5 \\ x & = & \frac{5}{7} \end{array}$$

Thus, the solution is $(\frac{5}{7}, -\frac{31}{14})$.

Exercises

Solve the systems using elimination.

1. $\begin{cases} x + y = 3 \\ 3x - 2y = -1 \end{cases}$
2. $\begin{cases} x + y = 7 \\ 2x - 3y = 4 \end{cases}$
3. $\begin{cases} 2x - 3y = 4 \\ 7x + 4y = -15 \end{cases}$
4. $\begin{cases} 2x - 3y = -1 \\ 5x + 2y = 45 \end{cases}$
5. $\begin{cases} 7x + 2y = 1 \\ 4x - y = 7 \end{cases}$
6. $\begin{cases} 3x - 2y = 1 \\ 16x + 12y = 11 \end{cases}$
7. $\begin{cases} 5a + 7b = 14 \\ 3a - 4b = -8 \end{cases}$
8. $\begin{cases} 3x - 10y = 32 \\ 6x - 20y = 64 \end{cases}$
9. $\begin{cases} 3x + 6y = 7 \\ 6x + 12y = 15 \end{cases}$
10. $\begin{cases} 10x + 3y = 159 \\ 3x + 10y = 166 \end{cases}$

11. $\begin{cases} 2x+5y &= 49 \\ 3x-2y &= -50 \end{cases}$

12. $\begin{cases} 4x+5y &= 10 \\ 7x+3y &= 6 \end{cases}$

13. $\begin{cases} 3x+2y &= 60 \\ 2x+3y &= 60 \end{cases}$

14. $\begin{cases} 4x-6y &= -96 \\ 10x+3y &= 120 \end{cases}$

15. $\begin{cases} 8x+5y &= -1 \\ 4x-10y &= 7 \end{cases}$

16. $\begin{cases} x-2y &= 2 \\ 2x-6y &= 3 \end{cases}$

17. $\begin{cases} x+y &= 7 \\ 2x+2y &= 14 \end{cases}$

18. $\begin{cases} 3x-4y &= 5 \\ 6x-8y &= 7 \end{cases}$