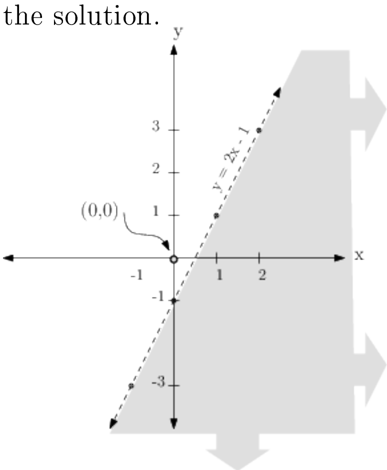
4.6 Linear Inequalities in 2 Variables

The graph of any linear inequality in two variables, such as $y < 2x - 1$, is going to be a half-plane containing infinitely many points, and the finite border of this half plane will be the linear equation associated with the linear inequality.

Example Sketch the graph of the solution set for $y < 2x - 1$.
The border of this solution, a half-plane, will be the line $y = 2x - 1$. However, no point on this line is a solution because y is strictly less than 2x - 1, and so the line is dashed to show that it is not part of the solution.



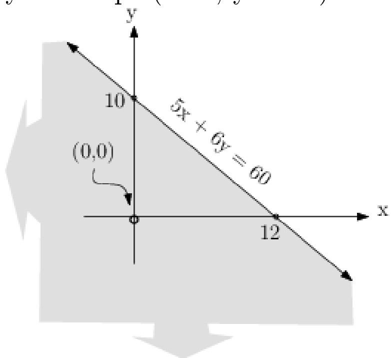
There are two choice for the solution once the border $y = 2x - 1$ (dashed) is drawn, each a half plane. To find out which is the solution, test an "easy" point which is clearly one one side of the line or the other side. If the point (0,0) is not on the line, it is always the "easiest" point. If the point is a solution, the half plane contains the point, and if the point fails to be a solution, the other side of the line is the half-plane solution.

Test the point (0,0).

$$\begin{aligned} Test\,(0,0): \quad y &< 2x - 1 \\ 0 &< 2(0) - 1 \\ False! \quad 0 &< -1 \end{aligned}$$

Thus the half plane without (0,0) is shaded to be the solution.

Example Graph the solution set for the inequality $5x + 6y \leq 60$.
This is an easy line to sketch by plotting the x-intercept (y=0, x = 12) and y-intercept (x=0, y = 10).



The line $5x + 6y = 60$ is included in the half-plane solution because the inequality $\leq$ allows equality. Testing an easy point, (0,0), we find

$$\begin{aligned} Test\,(0,0) \quad 5x + 6y &\leq 60 \\ 5(0) + 6(0) &\leq 60 \\ True! \quad 0 &\leq 60 \end{aligned}$$

So the side of the line including the point (0,0) is the solution set.

Systems of Linear Inequalities

Linear inequalities are used in a great many practical applications, and usually there is more than one. When more than one inequality must be true, we have a system.

Example Describe the limits of factory production with a system of linear inequalities. This factory overhauls trucks and cars, and it can only overhaul 20 per week combined.

We let x be the number of trucks overhauled and y be the number of cars overhauled. Thus

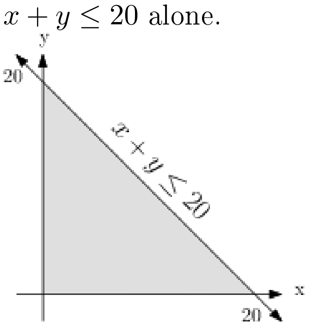
$$x + y \leq 20$$

We actually have some extra conditions here in this practical problem which are that x and y must be greater than or equal to zero. So, we have the system

$$\begin{cases} x + y \leq 20 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

The graph of this system makes more sense for the problem than the graph of

$x + y \leq 20$ alone.



Exercises Graph the following inequalities.

1. $2x - 3y > 12$
2. $4x + 5y \leq 20$
3. $x - 3y < 12$
4. $y \geq 2x - 3$

Graph the following systems.

5. $\begin{cases} x + y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$
6. $\begin{cases} x + 2y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$

7. A coffee shop sells decaf and regular cups of coffee. It can sell at most 500 cups on a particular day because there are only 500 paper cups. Let x be the cups of decaf sold, and let y be the cups of regular sold. Write a system decribing the possible sales for this day.
8. A car salesman can process at most 8 sales per day including used and new cars. Let x be the number of new cars sold, and let y be the number of new cars sold. Write a system describing the possible sales of cars for this salesman.