4.5 Equation of a Line

The equation of the line with slope m through the point (x_1, y_1) is

$$y = m(x - x_1) + y_1$$

Example What is the equation of the line through (1,-3) with slope m = 4 ?

$$\begin{aligned} y &= 4(x - 1) + (-3) \\ &= 4x - 4 - 3 \\ y &= 4x - 7 \end{aligned}$$

It is often useful to put the line into the form $y = mx + b$ for the answer.

Equation of a line through Two Points (x_1, y_1) (x_2, y_2)

The point slope form of a line needs a slope and a point. Given two points, either point will be fine, but it is necessary to find the slope first using the formula for the slope of a line between two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Then, using this slope m and one point, say (x_1, y_1), we use the point slope formula.

Example What is the slope of the line through the points (3,-4) (6,8) ?
First, find the slope.

$$m = \frac{8 - (-4)}{6 - 3} = \frac{12}{3} = 4$$

Then, use the point-slope formula.

$$\begin{aligned} y &= 4(x - 3) - 4 \\ &= 4x - 12 + 4 \\ y &= 4x - 8 \end{aligned}$$

Example What is the equation of the line through (-2,3) which is parallel to the line $3x + 2y = -11$?
We find the slope of the line $3x + 2y = -11$ first.

$$\begin{aligned} 3x + 2y &= -11 \\ 2y &= -3x - 11 \\ y &= \frac{-3}{2}x - \frac{11}{2} \end{aligned}$$

Thus, the slope of this line is $\frac{-3}{2}$.
Then, we use this slope and the point in the point-slope formula.

$$\begin{aligned} y &= \frac{-3}{2}(x - (-2)) + 3 \\ &= \frac{-3}{2}(x + 2) + 3 \\ &= \frac{-3}{2}x - 3 + 3 \\ y &= \frac{-3}{2}x \end{aligned}$$

Example The population of a county was 43,100 in 2000, and it rose to 56,500 in 2008. Assuming that population grows linearly, what is the projected population of the county in 2020?
To simplify matters, consider the year 2000 as x = 0, the year 2009 and x=9, and 2020 as x = 20. In short, x stands for the number of years past the year 2000. This simplifies the problem considerably. We consider x as the year past 2000, and y to be the population of the county, and so we have two ordered pairs: (0,43100) and (9,56500). We find the slope of the line through these two points.

$$\begin{aligned} m &= \frac{56500 - 43100}{9 - 0} \\ &= \frac{13400}{9} \approx 1488.8889 \\ m &= 1489 \end{aligned}$$

Note that we have rounded here and set $m = 1489$. This is an approximation, but approximations are fine and the best that can be done for such projections using available data.
Then, we use one point, and (0,43100) is certainly the easiest point to use.

$$\begin{aligned} y &= 1489(x - 0) + 43100 \\ y &= 1489x + 43,100 \end{aligned}$$

For an interpretation, the slope m = 1489 is the annual population growth in the county.
To project the population in year 2020, we replace x with 20 in this linear equation.

$$
\begin{aligned}
y &= 1489 \cdot 20 + 43,100 \\
&= 29,780 + 43,100 \\
&= 72,880
\end{aligned}
$$

Exercises

1. What is the equation of the line through the points (2, 3) (4, 7) ?

2. What is the equation of the line through the points (3, 3) and (-4, 6) ?

3. What is the equation of the line through the points (3, -7) (1, 13) ?

4. What is the equation of the line through the points (2.3, 6.5) (4.7, 81.4) ?

The population of a city is 1200 in 1990 (year 0), and the population of this city is 1300 in 2000 (year 10).

5. Model the population (y) of the city by the year after 1990 (x) using the information above to form a linear equation.

6. Using your model, predict the population of this city in 2008.

7. What is the equation of the line through the point (3,4) which is parallel to the line $y = -4x + 9$?

8. What is the equation of the line through the point (7, 6) parallel to the line $4x + 6y = 1$?