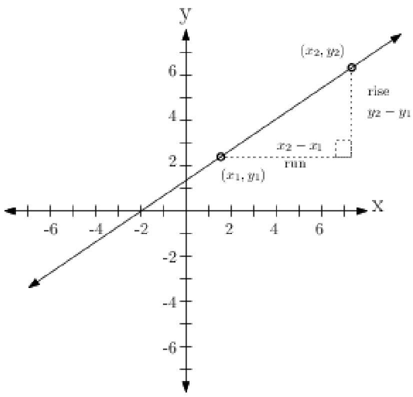
4.4 Slope of a Line

A line is completely determined by two different points.

The slope m of a line through the points $(x_1, y_1) = (x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



The slope, m, is also called the change in y with respect to the change in x, also called the rise over the run.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run} = \frac{change\, in\, y}{change\, in\, x}$$

Example Find the slope of the line through the points (3, 5) and (6, -1).

$$m = \frac{-1-5}{6-3} = \frac{-6}{3} = -2$$

Slope of a Horizontal Line

The slope is zero when the change in y is zero. This is always the case for a horizonal line.

Example What is the slope of the line through the points (4,3) and (-1, 3) ?

$$m = \frac{3-3}{-1-4} = \frac{0}{-5} = 0$$

The slope of a horizontal line is 0.
The equation of a horizontal line is of the form

$$y = b$$

For example, the equation of the horizontal line through the points (4,3) and (-1,3) is

$$y = 3$$

Slope of a Vertical Line

The slope of a vertical line is undefined.

Example Find the slope of a line through the points (2,5) (2, -1).

$$m = \frac{-1-5}{2-2} = \frac{-6}{0} \quad m\ is\ undefined$$

The equation of a vertical line is always for the form

$$x = a$$

For example, the equation of the vertical line through the points (2,5) and (2, -1) is

$$x = 2$$

Slope-Intercept Form of a Line

The equation of the line with slope m and y-axis intercept b is

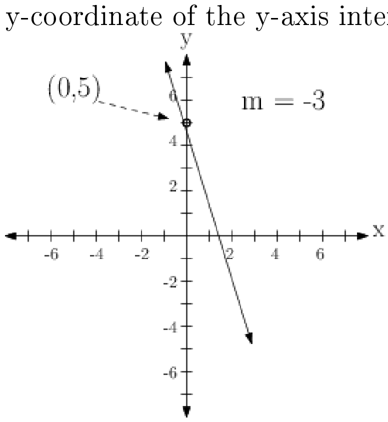
$$y = mx + b$$

This form also tells us that the way to find the slope of line and the y coordinate of the y-axis intercept is to solve for y, then read m and and b.

Example What is the slope of the line $6x + 2y = 10$?

$$\begin{array}{rcl} 6x + 2y & = & 10 \\ -6x & & -6x \\ \hline 2y & = & -6x + 10 \\ y & = & -3x + 5 \end{array}$$

Thus, the slope of the line is -3, and the y-coordinate of the y-axis intercept is 5.



Perpendicular Lines

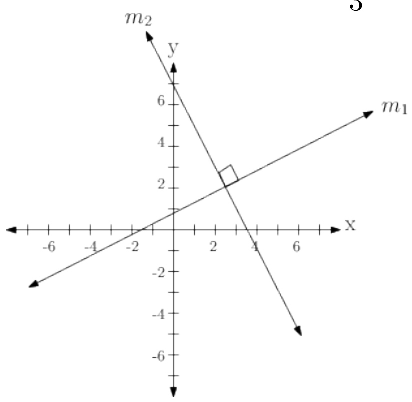
Perpendicular lines meet at right angles, and, with one exception, the product of their slopes is -1. The exception is the fact that vertical and horizontal lines are perpendicular to eachother. The slope of a vertical line is undefined, the slope of a horizontal line is 0, but we cannot and do not use the formula below in this special case.

If m_1 and m_2 are the slopes of two perpendicular lines–not vertical and horizontal lines–then

$$m_2 = \frac{1}{m_1}$$

This says that the slopes of lines perpendicular to eachother are negative reciprocals of eachother's slopes.

Example What is the slope of a line perpendicular to the line $y = \frac{2}{3}x + 5$?
We see that the slope of this line is $\frac{2}{3}$.
The negative reciprocal of $\frac{2}{3}$ is $\frac{-3}{2}$.



Example What is the slope of a line perpendicular to $y = -4x + 1$?
The slope of this line is -4. We can use the formula above.

$$m_2 = \frac{1}{m_1} = \frac{1}{-4} = -\frac{1}{4}$$

Exercises

Find the slope of the line through the following pairs of points.

1. (2, 3) (4, 11)
2. (-3, 1) (4, 20)
3. (0, 2) (5, -4)
4. (1,3) (1,-6)
5. (-3,5) (10,5)
6. (1,4) (11, -1)
7. (0, 32) (100, 212)
8. (6, 4) (-1, 18)
9. Find the equation of the vertical line that contains the point (5, 6).
10. Find the equation of the horizontal line containing the point (9, 12).
11. Find the equation of the line through (2, 4) with slope equal to zero.

Find the slope and y-axis intercept for the following lines.

12. 3x + y = 5
13. 4x - 2y = 9
14. 3y = 5x + 12
15. y = -2
16. x = 7
17. 3x - 2y = 8

Find the slope of any line perpendicular to the following lines.

18. y = 5x + 9
19. $y = \frac{-3}{5}x + 11$
20. $y = -3x + 8$
21. $3x + 8y = 4$
22. x = -1
23. y = 3