4.3 Linear Functions

The linear equation

$$y = mx + b$$

defines y as a function of x. We can be explicit about the "function" by using the notation

$$f(x) = mx + b$$

In either form, x is multiplied by m and then increased by b. It is convenient to use the $f(x)$ form when algebraically finding the functional value is important.

Evaluating Linear Functions

To evaluate a linear function, we mean finding an ordered pair solution. For all linear functions in two variables, there are infinitely many solutions. However, often when one variable is known, the other variable can be determined.

Example Consider the linear equations $y = 3x + 7$.
What is the value of y when x is 4?

$$y = 3 \cdot 4 + 7 = 12 + 7 = 19$$

Thus y = 19 when x = 4, and (4, 19) is an ordered pair solution.

Example Consider the linear equation $3x - 4y = 24$. Solve for y.

$$\begin{aligned} 3x - 4y &= 24 \\ -3x \quad & \quad -3x \\ \hline -4y &= -3x + 24 \\ \frac{-4y}{-4} &= \frac{-3x}{-4} + \frac{24}{-4} \\ y &= \frac{3}{4}x - 6 \end{aligned}$$

Example Is (-3,5) a solution to the equation $y = 2x + 11$?
We substitute x with -3, and y with 5, then see if equality is true.

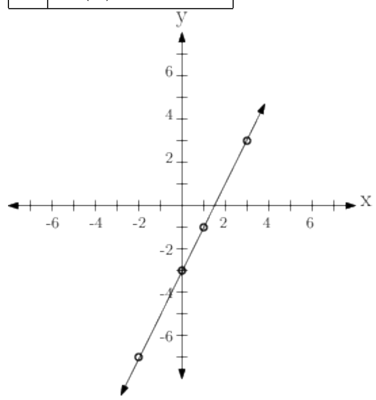
$$\begin{aligned} 5 &\stackrel{?}{=} 2(-3) + 11 \\ 5 &\stackrel{?}{=} -6 + 11 \\ 5 &= 5 \checkmark \end{aligned}$$

Graphing Linear Functions

The graph of a linear function on the Cartesian plane is a straight line. It is usual to use the x coordinate for the horizontal axis, and the y coordinate for the vertical axis. Two points are sufficient, but sometimes several points are plotted.

Example Plot the equation $y = 2x - 3$.
We make a table of values, and usually we generate the table by picking values of x and then calculating values of y.
This is almost always done when the equation is expressed in the form "y =".

x	y = 2x - 3
-2	2(-2) - 3 = -7
0	2(0) - 3 = -3
1	2(1) - 3 = -1
3	2(3) - 3 = 3

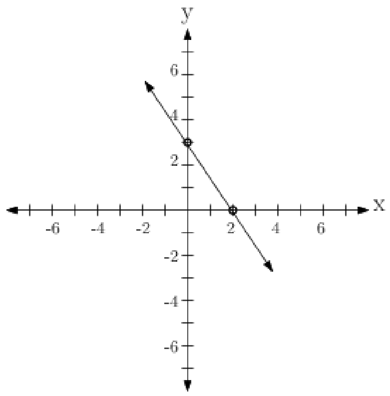


Example Plot the equation $3x + 2y = 6$ using the axis intercepts.
For equations of lines in which there is a constant, like 6 in this equation above, the axis intercept plotting method is quite nice. In this case, it is easy to set x = 0 and solve for y, then set y = 0 and solve for x. As a benefit, we get the two interesting points on the coordinates axes.

x	y
0	
	0

When x = 0, we solve $2y = 6$ which resluts in y = 3.
When y = 0, we solve $3x = 6$ which results in x = 2.

x	y
0	3
2	0



Example The order cost of having x cases of soda delivered is \$6 per case plus a \$5 delivery charge.

1. Write a function which determines the order cost given x cases of soda to be delivered.

$$f(x) = 6x + 5$$

2. What is the order cost for having 25 cases of pop delivered?

$$f(25) = 6(25) + 5 = 155$$

The order cost is \$155.

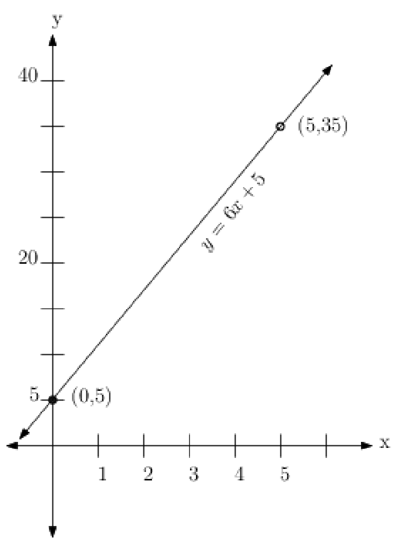
3. Graph this function.
Here we look at the function in the form

$$y = 6x + 5$$

Find two points on the line.

x	y
0	5
6	35

Plot the line.



Exercises

1. Graph $y = 2x-5$ by plotting three points.
2. Graph $y = x + 4$ by plotting two points.
3. Graph y = 5x-7 by plotting two points.
4. Graph y = -2x+6 by plotting two points.
5. Graph 2x+5y = 20 by plotting axis intercepts.
6. Graph 3x - 4y = 24 by plotting axis intercepts.
7. Graph y = 7 by plotting two points.
8. Graph x = -2 by plotting two points.
9. For $3x + 6y = 18$, solve for y.
10. For $5y - 2x = 7$, solve for y.