4.2 Functions

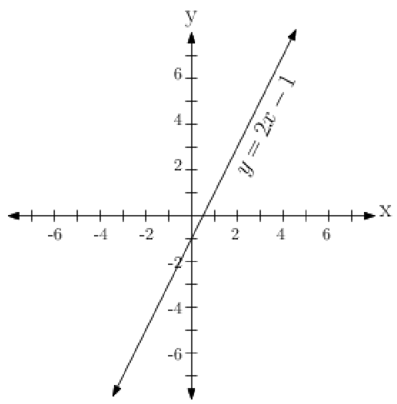
The equation $y = 2x - 1$ suggests that we have a formula for y given each value of x. In this sense, we say that x is the independent variable and y is the dependent variable because the value of y depends on x.

We can formalize this dependence on x using function notation.

$$f(x) = 2x - 1$$

This reads "f of x equals two x minus one". If we let $y = f(x)$, then we are back to our equation $y = 2x - 1$. In fact, these two forms are equivalent in a functional sense.

$$\begin{aligned} y &= 2x - 1 \\ f(x) &= 2x - 1 \end{aligned}$$



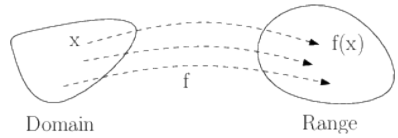
It is convenient to graph the equation $y = 2x - 1$, but it is convenient to use the notation $f(x) = 2x - 1$ in a functional sense. In this functional sense, "f" is used to do something to a number which, in this case, is to multiply a number by two then subtract one.

$$\begin{aligned} f(3) &= 2(3) - 1 = 5 \\ f(10) &= 2(10) - 1 = 19 \\ f(-7) &= 2(-7) - 1 = -15 \\ f\left(\frac{7}{3}\right) &= 2\left(\frac{7}{3}\right) - 1 = \frac{14}{3} - 1 = \frac{11}{3} \\ f(z) &= 2z - 1 \\ f(\heartsuit) &= 2\heartsuit - 1 \end{aligned}$$

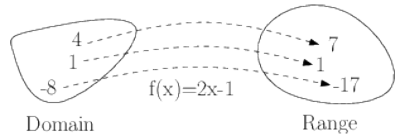
There are many ways to define functions, and it is traditional to use a letter like f, g, and h with an independent variable in the parenthesis.

$g(z) = 5z + 3$ defines g to be function which multiplies a number by five then adds three, yet $g(x) = 5x+3$ is the same definition because it does the same thing to a number. In defining functions, the variable used in the definition is usually x, but the variable is used only to define what the function does.

Definition A function $f(x)$ assigns a unique element from its range to each element x of its domain.



The natural domain for our function will be the set of real numbers or, perhaps, some subset of the real numbers. the range will be some subset of the real numbers. For example, the function $f(x) = 2x - 1$ is defined for every real number, so its domain is $\mathbb{R}$, the set of all real numbers.



Domain of a Real-Valued Function

The domain of a function is that subset of the real numbers for which the function is defined. $f(x) = 2x - 1$ is defined for all real numbers because any real number x can be multiplied by 2 then decreased by one, and the result is a real number. Problems come from division by zero because anything divided by zero is not a real number.

For example, the function $g(x) = \frac{1}{x}$ is not defined for x = 0 because $g(0) = \frac{1}{0}$, and 1/0 is not a real number. Division by any real number except for zero is OK, so the domain of g is $\{x | x \in \mathbb{R}, x \neq 0\}$ or simply $x \neq 0$.

This mathematical definition of a function's domain is also called the natural domain of a function. For many applications, it is practical to further restrict the domain of a function.

Example The amount of tax to be paid on purchases in Minnesota is 6.5% of the purchase price. Write the tax as a function of the purchase price x.

$$f(x) = .065x$$

The natural domain for this function is all real numbers. However, it does not make sense for a purchase price to be less than zero. Here, we reasonably restrict the domain ourselves to be $x > 0$.

What is the tax on a $135 purchase?
$f(134.95) = .065(134.95) = 8.77175$
Here again, in practical problems we try to be practical. Customers cannot easily pay $8.77175 in cash, so here we round to the closest penny. The tax on this purchase is $8.77 .

Exercises

Consider the functions
$f(x) = 6x - 4$,
$g(x) = x^2 + 3x - 1$, and
$h(x) = \frac{3x}{x+2}$
Compute the following.

1. f(5)
2. g(2)
3. h(-3)
4. g(-3)
5. h(20)
6. g(3.81)
7. h(492.6)

Consider the functions
$f(x) = x^2 + 5$ and
$g(x) = \frac{x+10}{2x+3}$

8. Calculate f(7).
9. Calculate g(2).

10. Consider the function $f(x) = \frac{1}{x-2}$. What value of x would have to be excluded from the domain of this function?
11. Consider the function $g(x) = \frac{1}{2x-3}$. What value of x would have to be excluded from the domain of this function?
12. The costs of running a particular bar include $220 in fixed costs (rent, payroll, etc.) and $1.10 per beverage sold. Express the cost of running this bar as a function of the number of beverages sold x.
13. The volume of a cube is a function of the length on its sides. Express the volume of a cube as the function of its side length s.
14. The mileage of a particular truck is 18mpg, but it loses 1 mpg for each ton of cargo, and it can hold a maximum of 8 tons. Express the truck's mileage as a function of its cargo weight x. What should be the domain of this function?