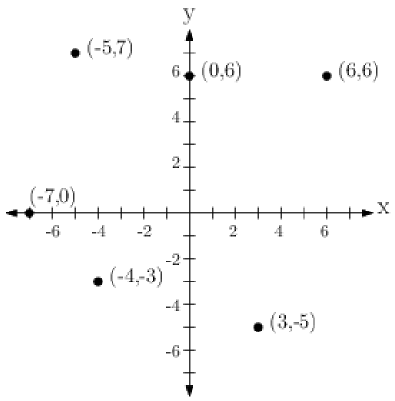
4. Linear functions and inequalities in 2 variables

4.1 Cartesian Plane

The Cartesian plane is also called the coordinate plane, the xy plane, or simply “The Plane”. “Cartesian” comes from Rene Descartes who applied coordinate algebra to the study of geometry. the x-axis and y-axis are perpendicular real number lines crossing at their zero coordinates which is called the origin, point (0,0) where x=0 and y = 0. Points on the plane are defined by ordered pairs, (x,y), where x and y each correspond to distances from the origin in the direction of the x-axis and y-axis respectively. The point (3,-5) is 3 units right and 5 units down from the origin. The point (-7,0) is 7 units left of the origin and 0 units up, so it lies on the x-axis. The point (0,6) lies 6 units up from the origin on the y-axis.



Findng Solutions to Equations on The Plane

The equation $y = 2x + 1$ has infinitely many solutions. However, if we know the value of one variable, we can sometimes find the value of the other variable.

Example Given the equation $y = 2x + 1$,if x = 1, find the value of y.

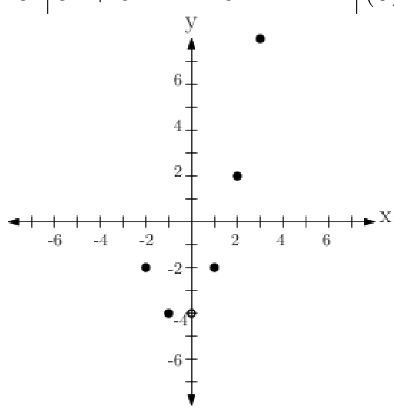
$$\begin{aligned} y &= 2x + 1 \\ y &= 2(1) + 1 \\ y &= 3 \end{aligned}$$

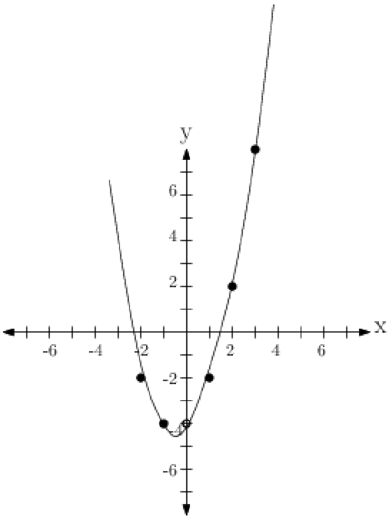
Thus, y = 3 when x = 1. We can plot this solution on the plane as the point (1,3).

In fact, for many equations we can find the corresponding value of y for each chosen value of x, and we can thus plot as many points as we like.

Example Given the equation $y = x^2 + x - 4$, find the values of y corresponding to x = -2, -1, 0, 1, 2, and 3. Then, plot the ordered pairs.
Here we will be creating six ordered pairs of values (x, y) which satisfy the equation $y = x^2 + x - 4$, and it is convenient to organize these ordered pairs in a table, then.

x	$y = x^2 + x - 4$	(x, y)
-2	$(-2)^2 + (-2) - 4 = -2$	(-2, -2)
-1	$(-1)^2 + (-1) - 4 = -4$	(-1, -4)
0	$0^2 + 0 - 4 = -4$	(0, -4)
1	$1^2 + 1 - 4 = -2$	(1, -2)
2	$2^2 + 2 - 4 = 2$	(2, 2)
3	$3^2 + 3 - 4 = 8$	(3, 8)





The points are plotted in the above-left graph. The curve $y = x^2 + x - 4$ is actually smooth and unbroken as shown in the above-right graph.

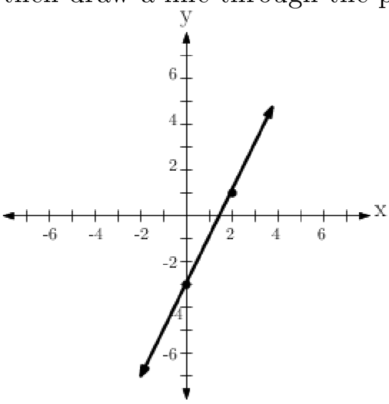
In sketching graphs of equations by hand, it helps to know something about the appearance of the graph in advance. Our simplest and most important graphs will be straight lines which can be nicely graphed by plotting only two points.

Example Graph the equation $y = 2x - 3$.

Two points are required, but which two points? The answer is any two points! Since any two points will do, it is wise to choose "easy" points, say for x = 0 and x = 2, then find the corresponding y values to each value of x.

x	y
0	-3
2	1

Plot these two points (0, -3) and (2, 1), then draw a line through the points.



Scaling and Labeling of Axes

It is easiest to start working with graphs labeled and scaled as the ones above with these characteristics.

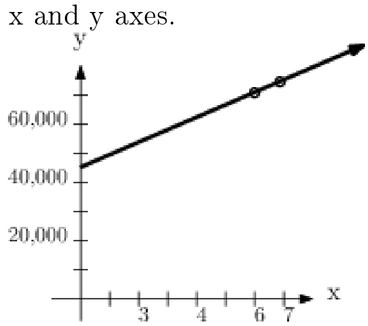
- x is the horizontal axis position
- y is the vertical axis position
- both axes cross at the origin (0, 0) in the center
- unit distances are the same on both axes
- tick marks on axes are one unit apart

In first working with graphs, these simplifying characteristics are usual. In fact, most graphing calculators have a "normal" graphing window with these characteristics.

However, it is often convenient or even essential to change all of these "standard" or "normal" graph characteristics. Such changes are almost always done to help understand a particular problem through graphing. (Also, such changes are sometimes done for deception!)

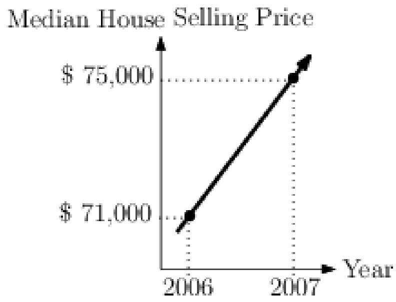
Example The median selling price of a house in Cloquet, Minnesota was \$71,000 in 2006, and the median price rose to \$75,000 in 2007. Let x be the number of years after the year 2000, and let y be the median selling price. If we plotted the two points (6,71000) and (7,75000) with the usual characteristics, the resulting graph would not be nearly impossible to see.

A good choice for such applied graphing problems is to use different scaling on the x and y axes.



Here a straight line pointing forward in time (and up in house price) is drawn through the two points. This is called a straight line approximation, and it is termed an approximation because reality rarely matches these mathematical models.

It is sometimes useful to entirely abandon all the "normal" graph characteristics. Here is a simple yet descriptive graph showing the same problem.



Exercises

1. Draw an xy plane, and plot the following points.
(0,0) (3,2) (-5,6) (4,4) (-2,-4)

2. Plot the points (3, 2) and (-4,1), then sketch a line through the two points.

3. Draw an xy plane, plot two ordered pair solutions (points) to the equation $y = 2x+1$, then sketch a line through the two points.

4. Draw an xy plane, plot two ordered pair solutions (points) to the equation $y = x + 2$, then sketch a line through the two points.

5. Draw an xy plane, plot two ordered pair solutions to the equation $y = 3$, then sketch a line through the two points. (HINT: y is always 3.)

6. Draw an xy plane, plot two ordered pair solutions to the equation $x = -5$, then sketch a line through the two points. (HINT: x is always -5.)

7. A VW Beetle was bought for $999.99 in 1969. This Beetle was stored in a garage, maintained yearly, and driven at most 500 miles a year only during nice summer days. It is worth $14,500 in 2009. Make a graph showing the increase in value over time.

8. A truck load of bananas was bought, and it was worth $850 on Monday. On Friday, the truckload of bananas was worth $300. Make a graph showing the decrease in value over time.