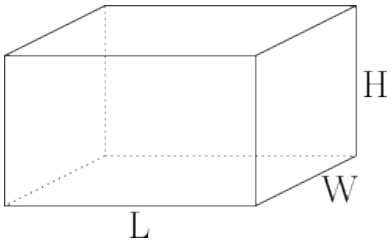
3.3 Solids

Rectangular Box with length L, width W, and height H

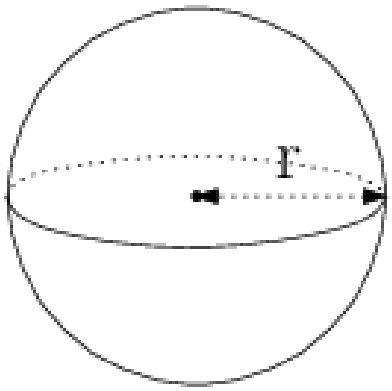


$$V = LWH$$
$$S = 2(LW + LH + WH)$$

Example What is the volume of a rectangular foundation excavation 40 feet long, 24 feet wide, and 8 feet deep?
$V = 40 \cdot 24 \cdot 8 = 7680\, ft^3$

Example What is the surface area of a 20 inch by 12 inch by 6 in cardboard mailing box?
$S = 2(20 \cdot 12 + 20 \cdot 6 + 12 \cdot 6) = 864\, in^2$

Sphere with radius r

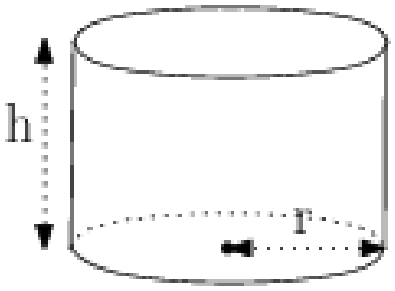


$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Example The Earth has a radius of 3963.2 miles. What is the volume and surface area of the Earth?
$V = \frac{4}{3}\pi(3963.2)^3 = 2.6075 \times 10^{11}\, mi^3 = 260,750,000,000.\, mi^3$
$S = 4\pi(3963.2)^2 = 197,380,000.\, mi^2$

Right Circular Cylinder with radius r and height h

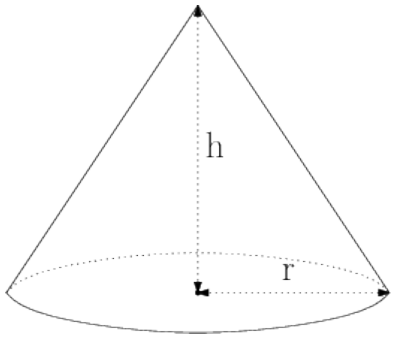


$$V = \pi r^2 h$$

$$S = 2\pi r^2 + 2\pi rh$$

Example What is the surface area and volume of a steel drum 3 feet high and 10 inches in radius?
The units must be the same. Cubic inches will be fine, so 3 ft = 36 in, and
$V = \pi(10)^2 36 = 3600\pi\, in^3$ or about $11,310\, in^2$
$S = 2\pi(10)^2 + 2\pi \cdot 10 \cdot 36 = 920\pi\, in^2$ or about $2890.3\ in^2$

Right circular Cone with radius r and height h

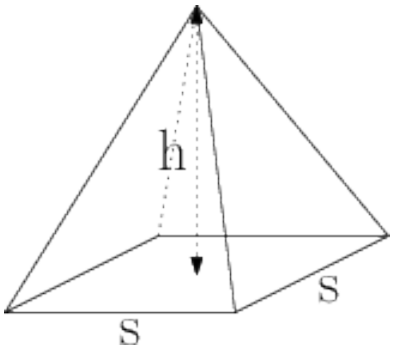


$$V = \frac{1}{3}\pi r^3 h$$

Example A conical pile of road salt is 21 feet high, and the radius of the circular base is 15 feet. What is the volume of the salt?
$V = \frac{1}{3}\pi(15)^3 21 = 23,625\pi\, ft^3$ or about $74,220.\, ft^3$

Road salt weighs 70 pounds per cubic foot. (Another name for weight per unit of volume is density.) How much does the conical pile of salt above weigh?
Weight = Volume $\cdot$ Density = $74,220 \cdot 70 = 5,195,400\, lb$

Right Pyramid with height h and base length s



$$V = \frac{1}{3}s^2h$$

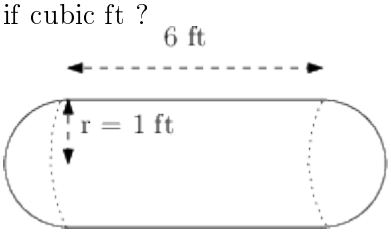
Example: Cleops' Pyramid was built in 2589-2566 BC. It has been heavily eroded as well as plundered for building materials (to build Cairo), but its original dimensions are known to be 482 feet high with a side length of 756 feet. What is the original volume of Cheops' pyramid?

$$V = \frac{1}{3}(756)^2(482) = 91,826,784\, ft^3\,.$$

If the density of limestone is $160\, lb/ft^3$, what would be the weight of Chep's pyramid assuming that it is a solid?
Weight = Volume $\cdot$ Density = $91,826,784 \cdot 160 = 14,692,285,440\, lb$ which is about 7.3 million tons .

Exercises

1. Find the volume of a sphere with a radius of 10 feet.
2. Find the surface area of the Moon in square miles. The radius of the Moon is 1080 miles.
3. A meter was originally defined to be 1 ten-millionth of the distance from the equator to the North Pole. Approximate surface area of The Earth in square kilometers by assuming that The Earth is a sphere.
4. Find the volume of a cylinder having height 10 feet and a radius of 3 feet.
5. An cylindrical oil can is 25 cm high and has a radius of 6 cm. What is the volume of this can in liters? 1 liter equals $1000 cm^3$.
6. A propane tank is made from an open steel cylinder 6 ft long with a radius of 1 ft, and the ends are capped with steel hemispheres each with a radius of 1 ft. What is the volume of this propane tank if cubic ft ?



7. A grain silo is 40 ft high with a radius of 8 feet, and it is topped with a hemisphere. What is the volume of the silo in cubic feet?