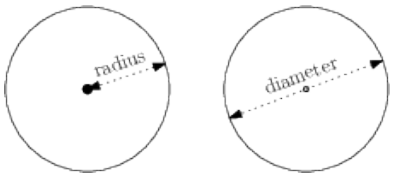
3.2 Plane figures

Plane figures that we consider here are the circle and polygons. Polygon means "many sides", and we mean a closed plane fiigure drawn with connected line segments. Polygons include most familiar plane figures such as the square, triangle, rectangle, and pentagon.

Circle

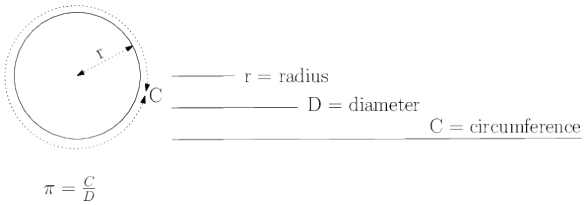
A circle is the set of points a fixed distance from a common center. The fixed distance from the circle's center to points on the circle is called the radius. The distance across the circle at its widest part is called the diameter



$$\begin{aligned} Radius &= \frac{1}{2} \cdot Diameter \\ Diameter &= 2 \cdot Radius \end{aligned}$$

The ratio of the circumference to the diameter is defined to be π.

$$\pi = \frac{Circumference}{Diameter}$$



π is an irrational number, close to 3.14, closer to 3.14159 . You should leave your answers to problems with a π to show an exact answer, yet you should use a decimal approximation for applied problems in which it is clear that you need an approximation.

Let r be the radius, D the diameter, and C the circumference of the circle.

$$\begin{aligned} C &= \pi D \\ C &= 2\pi r \end{aligned}$$

Example What is the circumference of a pizza with a radius of 6 inches?

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \cdot 6 \\ &= 12\pi \end{aligned}$$

The circumference is thus $12\pi\, in$ or about 37.7 in by approximating π to be 3.14 .

The area of a circle is πr^2.

$$Area_{\bigcirc} = \pi r^2$$

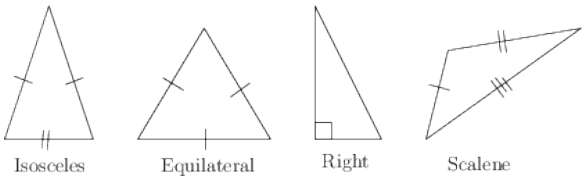
Example What is the area of a 2 meter disk?

$$\begin{aligned} A &= \pi r^2 \\ &= \pi (2)^2 \\ &= 4\pi \end{aligned}$$

The units of length are meters, so the units of area are in meters squared or m^2: $A = 4\pi\, m^2$. A decent approximation to the area of the 2 meter disk is made using 3.14 m for π: $A \approx 4(3.14)\, m^2 = 12.56\, m^2$.

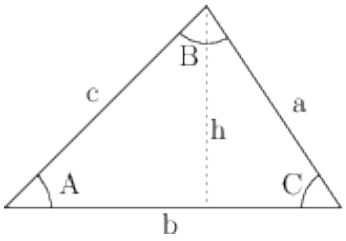
Triangles

A triangle has three sides and three angles. There are some common classifications of triangles.



The notation above indicates when sides are equal if they have the same number of slashes, and a different number of slashes shows that the sides are unequal. Note that a right triangle can be isosceles or scalene.

Every triangle can be considered as having a base and a height relative to the base. It is also useful to label the sides and angles and consider these angle and side labels as measurements with values in degrees and length.



h is the altitude

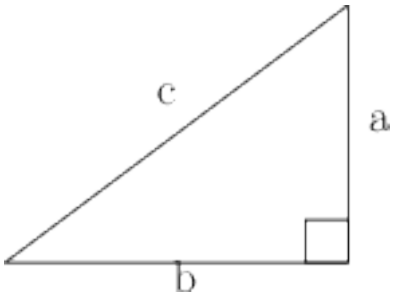
b is the base

The area of a triangle is one half of the product of the base b and the altitude h (height.)

$$\triangle_{Area} = \frac{1}{2}bh$$

Right Triangles and The Pythagorean Theorem

A right triangle implies a special relationship between the lengths of its three sides. The longest side opposite the right angle is called the hypotenuse. We typically label the length of the hypotenuse as c, then label the two sides adjacent to the right angle as a and b.

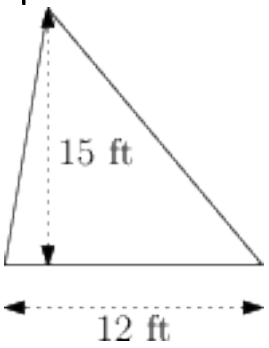


The Pythagorean theorem is that

$$a^2 + b^2 = c^2$$

This relationship between the lengths of the three sides hold for all right triangles. Further, if this relationship $a^2 + b^2 = c^2$ holds, we have a right triangle

Example Find the area of the triangle shown.



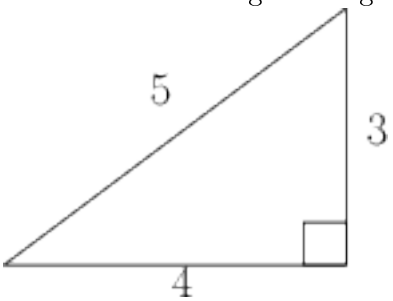
$$\begin{aligned} \triangle_{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2}(12)(15) \\ &= 90 \end{aligned}$$

Thus, the area of the triangle is 90 ft^2. Note that units for area will always be squared.

Example Find the area of a triangle with sides of length 5 ft, 12 ft, and 13 ft.
Our formula for the area of a triangle requires us to know the base and corresponding height of a triangle, and this is not necessarily available given the lengths of three sides. However, in this case we have

$$\begin{aligned} 5^2 + 12^2 &= 13^2 \\ 25 + 144 &= 169 \end{aligned}$$

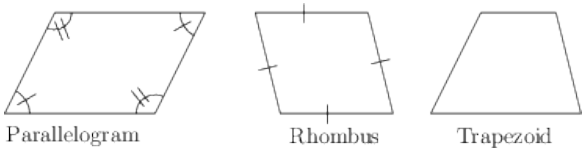
and so we have a right triangle.



The base is 4 and the height is 4, so $A = \frac{1}{2} \cdot 4 \cdot 3 = 6$.
The area of the triangle is 6 ft^2.

Quadrilaterals

A quadrilateral is a four-sided polygon. There are several common classifications of quadrilaterals.

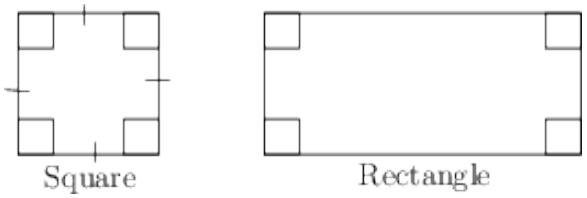


Opposite sides are parallel in a parallelogram, and this results in diagonal angles being the same. A Rhombus happens to also be a parallelogram. A trapeziod has two parallel sides

opposite eachother. Note that these definitions permit every rhombus to be called a parallelgram, and every rhombus and parallelogram can also be called a trapezoid. It is usual, however, to apply the strongest classification possible in order to say as much as can possibly be said about a figure. Note that opposite sides of a parallelogram are the same length, a fact not shown in the figure, and this fact is a consequence from opposite sides being parallel.

Quadrilaterals have four sides, and they can look unusual verses a rectangle, square, rhombus, quadrilateral, or trapezoid as one is more familiar with.

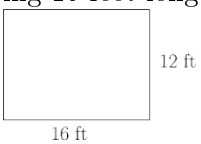
A rectangle's angles are all 90 degrees, and a square is a rectangle with all sides the same length.



The area of a rectangle is the length times the width.

$$Area_{Rectange} = L \cdot W$$

Example What is the area of a room measuring 16 feet long and 12 feet wide?



$$\begin{aligned} A &= L \cdot W \\ &= 16 \cdot 12 \\ &= 192 \end{aligned}$$

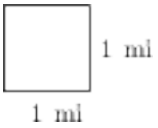
Thus, the area is $192\, ft^2$.

A square is a rectangle, so the formula for the area of a rectangle applies. However, it is usual to label the sides of a square with the letter s and consider a special formula.

$$Area_{square} = s^2$$

Example What is the area in feet of a square mile. A mile has 5280 ft.

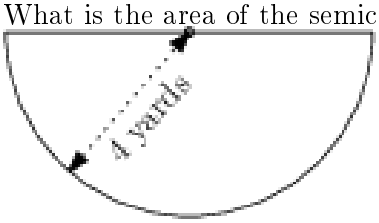
$$\begin{aligned} A &= s^2 \\ &= (5280)^2 \\ &= 27878400 \end{aligned}$$



Thus, the area of a square mile in feet is $27,878,400\, ft^2$.

Exercises

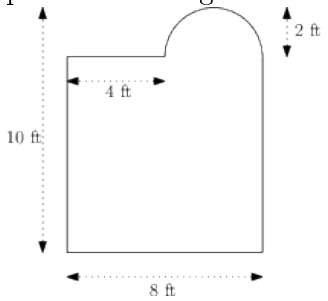
1. The diameter of a circle is 25 feet. What is the radius of the circle?
2. Find the circumference of a circle with a radius of 3 meters.
3. Find the area of a circle with a diameter of 8 inches.
4. What is the area of the semicircle here?



5. A rectangle has length 30 inches and width 12 inches. What is the perimeter of this rectangle?
6. What is the area of a rectangle 12 inches long and 7 inches wide?
7. A rectangle has a length of 5.2 cm and a width of 4 cm. What is the area of this rectangle?
8. The base of a triangle is 13.6 ft long, and the altitude is 4.3 ft. What is the area of the triangle?
9. The three sides of a right triangle have lengths 13, 12, and 5 inches. What is the area of the triangle?
10. How many square feet of carpet is needed to cover a rectangular room measuring 24 feet by 16 feet?
11. The two shorter sides of a right triangle are 2.3 ft and 6.7 ft long. What is the length of the hypotenuse?

12. The hypotenuse of a right triangle is 23 inches long, and one of the other sides is 10 inches long. What is the length of the remaining side?

13. What is the area of the figure below composed of rectangles and a semicircle?



14. What is the area of the figure below composed of a triangle and a semicircle?