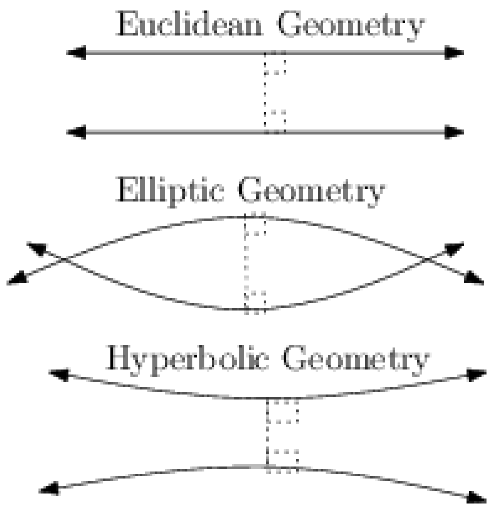
3. Geometry

Geometry is about the relationship between points, lines, plane figures, solids, and space. The subject of geometry is associated with the rigors of geometric proof, yet we focus here on some descriptive and practical aspects of geometry which are necessary for applications of algebra.

We apply here Euclidean geometry named so after Euclid for his book "Elements" from the 3d century B.C. Euclidean geometry is characterized by Euclid's 5th postulate, the parallel postulate, which defines parallel lines as being the same distance from eachother out to the reaches of infinity. This is the geometry we are familiar with and expect, yet there are alternatives.

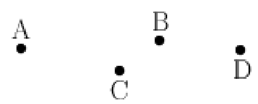


Elliptic and hyperbolic geometry arose as alternatives to Euclidean geometry about 200 years ago. In elliptic geometry, two lines might appear to be parallel at some point, yet the lines eventually meet and cross twice. This is not so unfamiliar an idea as this is exactly what happens for any two different straight lines on a sphere which are each great circles. Special methods are needed for navigation on the globe. With hyperbolic geometry, there are infinitely many different parallel lines to any particular line, and any pair of parallel lines do not meet but diverge apart to infinity toward their ends.

These alternatives to Euclidean geometry are not simply musings over abstract mathematical ideas. There are reasons to suspect that the universe is truly described by elliptic or hyperbolic geometry, and there are serious efforts made to find out. For practical purposes on Earth, however, Euclidean geometry seems to hold up well in describing the geometry of real things.

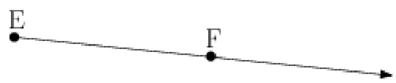
3.1 Points, Lines, and Angles

A point is a location without dimension, but we usually make a nice solid round dot to show one with the udnerstanding that the location itself is within the dot. Points are usually identified with capital letters like A, B, C, and D.



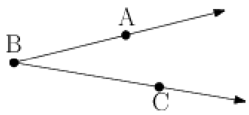
We define a line to be straight forever, in both directions, reaching out to the infinite at both ends. For a working formal definition of a line, a line is a set of points so that the shortest distance between any two points on the line is over a path entirely on the line itself. This is opposed to something curved which we will call a curve. In fact, two different points define a unique line–any two points on the line. If points A and B are on a line, we can describe the line using these two points with an arrown on top: $\overleftrightarrow{AB}$.

A line segment is defined by its two endpoints as the piece of line between the endpoints, and it is the shortest path between the endpoints. The notation is a small line segment over the two points. $\overline{CD}$ means the line segment with endpoints A and B.



A ray is half a line, and it is determined by two points. The ray starts at its initial point and passes through another point. The notation $\overrightarrow{EF}$ shows that E is the inital point of the ray passing through point F.

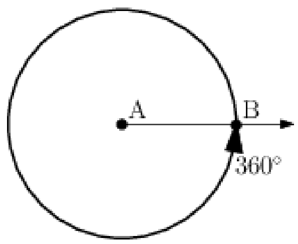
An angle is the intersection of two rays with a common initial point, and the common initial point of the angle is called the vertex. The angle $\measuredangle ABC$is the intersection of rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$.



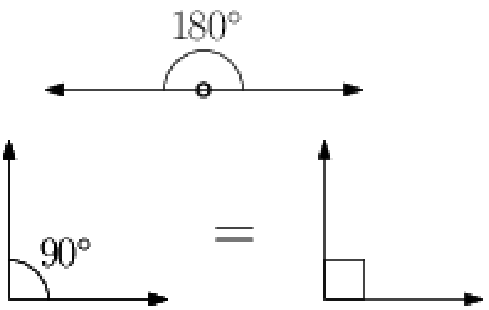
Now, in the Geometry of angles, $\measuredangle ABC$ is the same as $\measuredangle CBA$. We are talking here about the intersection of two rays which is a geometrical object.

Measurement of Angles

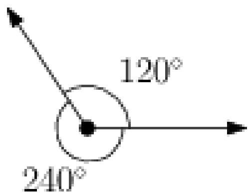
The measurement of an angle is a number attached to a unit like degrees. All angle measurements are based on a standard reference. For degrees, there are 360 degrees in a full circle of rotation.



Then, there are 180°in a flat angle and 90° in a right angle. The right angle is commonly shown by putting a square in the angle with one corner on the vertex.



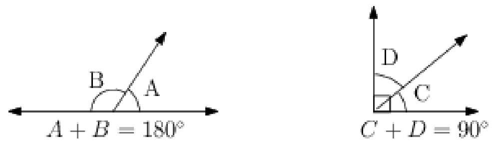
While the angle itself is two rays with a common vertex, the angle can be measured in two ways.



Unless indicated, the smaller angle is usually taken to be the measure of an angle.

Complementary and Supplementary Angles

Supplementary angles add up to 180°. Complementary angles add up to 90°.



Example What is the measure of the complementary angle to one measuring 23°?

$$\begin{aligned} x + 23 &= 90 \\ -23 & \quad\ -23 \\ x &= 67 \end{aligned}$$

The complementary angle is 67°.

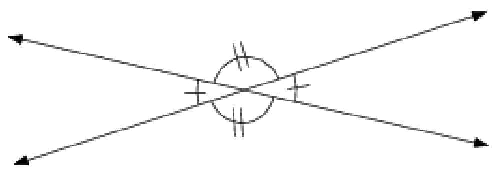
Example What is the supplementary angle to 111°?

$$\begin{aligned} x + 111 &= 180 \\ -111 & \quad\ -111 \\ x &= 69 \end{aligned}$$

The supplementary angle is 69°.

Vertical Angles

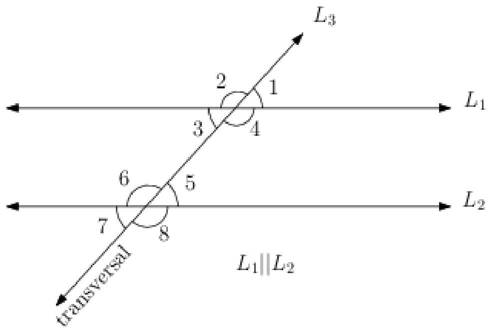
Vertical angles are the angles opposite of eachother formed by two intersecting lines. The measurements of vertical angles are the same.



Here we identify angles having the same measurement by drawing a number of slashes through the angle arc. Note that there are two pairs of vertical angles above.

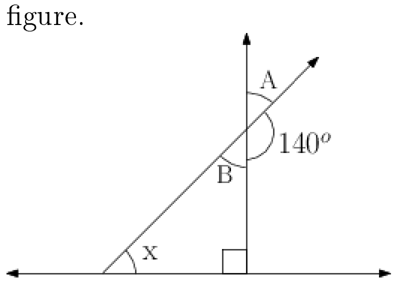
Transversal of parallel lines

A transversal of two parallel lines intersect both lines and forms 8 angles. Every possible pair of angles either has the same measurements or are complementary.



- $\angle 1 = \angle 3$, $\angle 5 = \angle 7$, $\angle 2 = \angle 4$, and $\angle 6 = \angle 8$ because vertical angles have the same measurement.
- $\angle 1 = \angle 5$,$\angle 2 = \angle 6$, $\angle 3 = \angle 7$, and $\angle 4 = \angle 8$ because corresponding angles have the same measurement.
- $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$ because alternate exterior angles have the same measurement.
- $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$ because alternate interior angles have the same measurement.

Example Find the value of angle x in the figure.

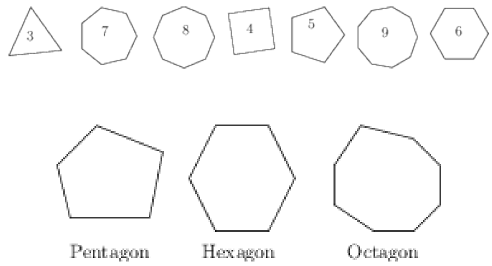


Angle B must be 40^obecause B is supplementary to a 140^oangle.
Then

$$\begin{aligned} x + B + 90^o &= 180^o \\ x + 130^o &= 180^o \\ x &= 50^o \end{aligned}$$

Polygons

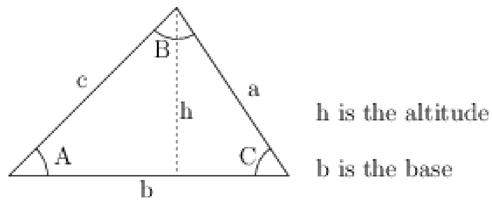
Polygon means "many sides". There are established names for many polygons based on the number of sides. To be understood, it is best to explain how many sides using "n-gon" for the number of sides n if there is an uncommon number of sides.



These are some of the common polygons: a pentagon, hexagon, and octagon. The triangle and rectangle are already familiar.

If the interior angles are the same and the lengths of the sides are the same, the polygon is called regular. A regular triangle is equilateral, and a regular quadrilateral is a square.

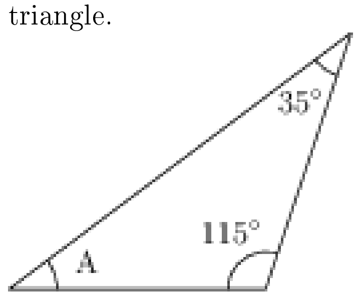
Angles and Triangles



The sum of the angles inside every triangle (A + B + C) is equal to 180 degrees.

$$A + B + C = 180°$$

Example Find the missing angle in the triangle.

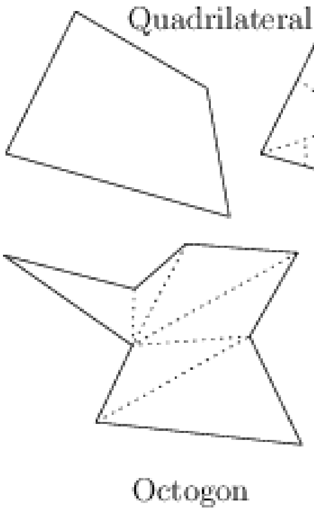


$$\begin{aligned} A + 115 + 35 &= 180 \\ A + 150 &= 180 \\ A &= 30 \end{aligned}$$

Thus, $A = 30°$.

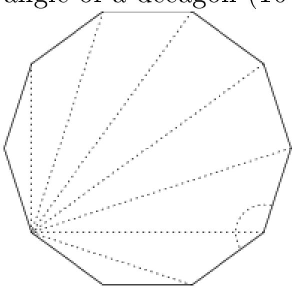
Interior Angles of a Polygon

The sum of angles inside a quadrilateral is 360 degrees, any quadrilateral. This fact is clear from adding up the four 90°angles inside a rectangle. For any plane polygon having 4 or more sides, the sum of interior angles can be viewed as the sum of interior angles of triangles partitioning up the polygon.



Each triangle has 180°for its interior angles, so it is easy to see that a quadrilateral has a sum of $2 \cdot 180° = 360°$for its interior angles. Similarly, an octogon is partitioned into 6 triangles, so its sum of interior angles is $6 \cdot 180° = 1080°$. For a regular polygon, every interior angle is the same.

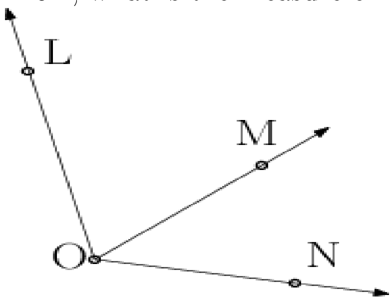
Example What is the measure of an interior angle of a decagon (10 sided polygon) ?



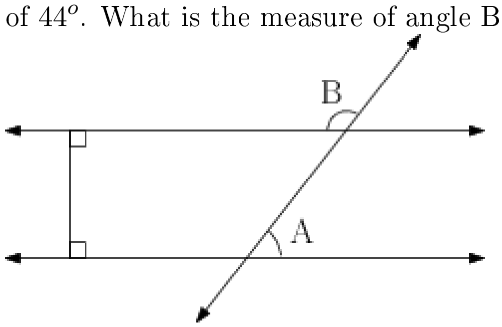
There are 8 interior triangles, so the sum of interior angles of the decagon is $8 \cdot 180° = 1440°$. Each of the 10 decagon angles are the same, so the measure of one regular decagon interior angle is $\frac{1440°}{10} = 144°$.

Exercises

1. What is the complement of a 63° angle?
2. What is the supplement of a 47° angle?
3. Given that $\sphericalangle MON = 55°$ and $\sphericalangle LON = 120°$, what is the measure of $\sphericalangle LOM$?



4. A triangle has two angles measuring 32^o and 47^o. What is the measure of the remaining angle?
5. A right triangle has an angle measuring 53^o. What is the measure of the other acute angle?
6. What is the sum of the interior angles of a hexagon?
7. What is the measure of an interior angle of a regular 9-gon?
8. A quadrilateral has interior angles measuring 55^o, 61^o, and 103^o. What is the measure of the remaining interior angle?
9. In the figure below, angle A has a measure of 44^o. What is the measure of angle B ?



10. What is the measure of an interior angle of a regular 13-gon?