# 2.6 Absolute Value Equations and Inequalities

The way to take the absolute value of a number is to make the number positive if it is negative or, otherwise, write down the number.

$$\begin{aligned} |-3| &= 3 \\ |3| &= 3 \end{aligned}$$

This is fine because we know whether a number os negative or not. With absolute value equations, we are being asked the question: What number can we replace the variable with to make the equation true? For the equation $|x| = 3$, we see above that x could be 3 or -3 because $|3| = 3$ and $|-3| = 3$. This is the case for absolute value equations where the absolute value of some linear expression in one variable is equal to a positive number; there will be two equations to solve.

$$|\heartsuit| = D$$
$$\heartsuit = D \qquad \heartsuit = -D$$

**Example** Solve $|x + 6| = 2$.

There are two separate linear equations to solve:

$$\begin{aligned} x + 6 &= 2 \\ -6 & \quad\ -6 \\ \hline x &= -4 \end{aligned}$$

$x + 6 = -2 - 6$
$-6$height

x = -8

Since -4 and -8 are two solutions, we express the solution set: $\{-4, -8\}$.

There are two special cases to consider.

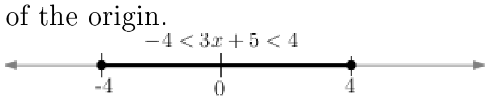
1. If we have the case $|\heartsuit| = 0$, we simply solve the equation $\heartsuit = 0$.
2. If we have the case $|\heartsuit| = \textit{Negative Number}$, the solution set is empty or $\emptyset$.

**Example** Solve $|2x + 7| = 0$.

$$\begin{aligned} 2x + 7 &= 0 \\ -7 & \quad\ -7 \\ \hline 2x &= -7 \\ \frac{2x}{2} &= \frac{-7}{2} \\ x &= \frac{-7}{2} \end{aligned}$$

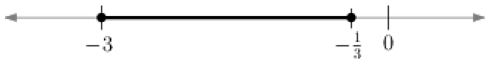
**Example** Solve $|3x + 5| \le 4$

This means that $3x + 4$ is within 4 units of the origin.



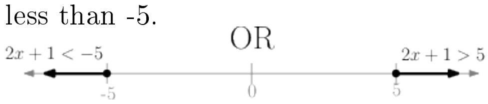

$$|3x + 5| \le 4$$

$$\begin{array}{ccccc} -4 & \le & 3x + 5 & \le & 4 \\ -5 & & -5 & & -5 \\ \hline -9 & \le & 3x & \le & -1 \\ \frac{-9}{3} & \le & \frac{3x}{3} & \le & \frac{-1}{3} \\ -3 & \le & x & \le & \frac{-1}{3} \end{array}$$



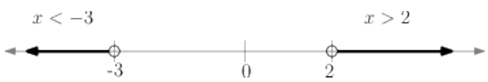

**Example** Solve $|2x + 1| > 5$

This means that $2x + 1$ is over 5 units away from the origin, greater than 5 OR less than -5.




We solve each inequality separately, then we union the results because either is a solution.

$$\begin{array}{ccc|ccc} 2x + 1 & < & -5 & 2x + 1 & > & 5 \\ 2x & < & -6 & 2x & > & 4 \\ x & < & -3 & x & > & 2 \end{array}$$

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**Exercises** Solve the following.

1. $|x + 4| = 10$
2. $|3 - x| = 1$
3. $|5 - 2x| = 7$

4. $|2x + 5| = 17$
5. $|4 - 3x| = 10$
6. $|x + 3| \leq 9$
7. $|2x - 1| \leq 5$
8. $5 + |3 - 2x| < 12$
9. $|x + 4| > 6$
10. $|3 - x| \geq 1$
11. $|2x - 3| > 3$