2.5 Linear Inequalities

Solving linear inequalities is alost the same process as solving linear equations. You can add the same thing to both sides.

$$A = B \quad \Rightarrow \quad A + C = B + C$$

Example Solve $x + 5 < 8$

$x + 5 < 8 - 5$

-5height

x $<$ 3

The solution $x < 5$ is an infinite set. It can be expressed in set form as $\{x|x < 5\}$which reads as "x such that x is less than 5". The vertical slash "|" is a shortcut to mean "such that". The solution can also be graphed on the real number line.

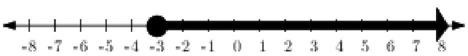
Another way to express this solution is in interval form. Consider that the numbers on the entire number line are between $-\infty$ and ∞.

Then, the interval $(-\infty, 5)$ is the set of number between $-\infty$ and 5.

Example Solve $3x + 4 \geq 2x + 1$.

$$\begin{array}{rcl} 3x+4 & \geq & 2x+1 \\ -2x & & -2x \\ \hline x+4 & \geq & 1 \\ -4 & & -4 \\ \hline x & \geq & -3 \end{array}$$

The other forms to express the solution $x \geq 3$ are similar to those for the strict inequalities $>$ and $<$. The solution expressed as an interval is $[3, \infty)$. The graph of the solution on the number line has a filled in circle to show that the number -3 is included in the solution.



There are four forms of inequalities.

$<$	Less than
$>$	Greater than
$\leq$	Less than or Equal to
$\geq$	Greater than or Equal to

Multiply or Dividing by a Negative

Inequalities with these symbols are solved in the same fashion with one special change from solving equations. When multiplying or dividing by a negative on both sides of an inequality, the direction of the inequality switches.

Example

$$\begin{aligned} -2x &< 7 \\ \frac{-2x}{-2} &> \frac{7}{-2} \\ x &> \frac{-7}{2} \end{aligned}$$

Example

$$\begin{aligned} -7x &\leq \frac{-1}{4} \\ \frac{-1}{7}(-7x) &\geq \frac{-1}{7} \cdot \frac{-1}{4} \\ x &\geq \frac{1}{28} \end{aligned}$$

Example

$$\begin{aligned} \frac{-3}{5}x &> \frac{2}{3} \\ \frac{-5}{3} \cdot \frac{-3}{5}x &< \frac{-5}{3} \cdot \frac{2}{3} \\ x &< \frac{-10}{9} \end{aligned}$$

Example

$$\begin{aligned} -\frac{3}{4}x &\geq -5 \\ \frac{-4}{3} \cdot -\frac{3}{4}x &\leq \frac{-4}{3} \cdot (-5) \\ x &\leq \frac{20}{3} \end{aligned}$$

Compound Inequalities

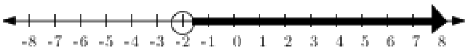
A the solution to a compound inequality is the intersection or union of two or more ordinary inequalities. The solution to two inequalities joined by OR is the solution to either one of them.

Example Find the solution to $2x > -1$ OR $x + 3 \leq 6$.

$$\begin{aligned} 2x &> -4 \\ \frac{2x}{2} &> \frac{-4}{2} \\ x &> -2 \end{aligned}$$

$$\begin{aligned} x+3 &\leq 6 \\ -3 \quad & \quad -3 \\ x &\leq 3 \end{aligned}$$

The graph of $x > -2$:



The graph of $x \leq 3$:

The graph of $(x > -2) \bigcup (x \leq 3)$:

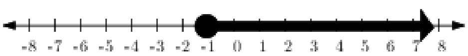
The solution as an interval is $(-\infty, \infty)$ or simply $\mathbb{R}$, the set of all real numbers.

Example Find the solution to $3 - 2x \leq 5$ AND $4x + 1 < 13$.

$$\begin{aligned} 3-2x &\leq 5 \\ -3 \quad & \quad -3 \\ \hline -2x &\leq 2 \\ \frac{-2x}{-2} &\geq \frac{2}{-2} \\ x &\geq -1 \end{aligned}$$

$$\begin{aligned} 4x+1 &< 13 \\ -1 \quad & \quad -1 \\ \hline 4x &< 12 \\ \frac{4x}{4} &< \frac{12}{4} \\ x &< 3 \end{aligned}$$

The graph of $x \geq -1$:



The graph of $x < 3$:

Then, the graph of $(x \geq -1) \bigcap (x < 3)$is where they overlap:

Exercises Solve the following. Graph the solution and express as an interval or intervals.

1. $x + 6 > -2$
2. $3x \leq 12$
3. $-5x > 35$
4. $2x + 7 < 8x + 3$
5. $-4x \leq 32$
6. $-2x \geq -18$
7. $3 - 5x \geq -22$
8. $3x - 5 < -2x + 5$
9. $7 - 2x < 3x - 3$
10. $-5 < 2x - 9 < 7$
11. $-4 < x - 2 < 1$
12. $7 \geq 5 - 2x \geq 1$
13. $3x - 2 > x - 4$ OR $7x - 5 < 3x + 3$
14. $3x < 4$ AND $x + 2 > -1$