2.4 Applications

A simple yet effective model for running a small retail business calls for looking at daily profit where profit equals revenue minus cost. There are two kinds of costs to consider in this model: fixed costs and variable costs. Fixed costs include utilities, payroll, rent, and other items whichare the same irregardless of the amount of sales. Variable costs are from the cost of good sold, and the variable is the number of goods sold. It is assumed that the selling price of goods is greater than the cost of goods to the retailer, and so revenue increases as the number of good sold increases. There is a certain number of sales in which the profit is zero, the break-even point. Sales greater than this break-even point results in a positive profit, but sales less than the break-even point results in a loss. The general equation for profit involves only one variable, x, in which x is the number of goods sold.

$$Profit = Revenue - Cost$$

Example Suppose that a pizza parlor sells pizzas for $12, the cost of making a pizza is $5, and the fixed cost for running the pizza parlor is $100 per day. We want to find the break even point.

Revenue Per Sale is 12, Cost Per Sale is 5, and Fixed Cost is $100. Let x be the number of pizzas sold.

$$Profit = 12x - 5x - 100$$

To find the break even point, set Profit equal to zero, then solve for the variable.

$$\begin{aligned} 0 &= 12x - 5x - 100 \\ 0 &= 7x - 100 \\ +100 & \quad +100 \\ 100 &= 7x \\ \frac{100}{7} &= x \\ x &\approx 14.29 \end{aligned}$$

In this practical problem, we do not sell fractions of pizzas, so we want a non-negative integer answer. 14 is closest to 100/7, yet the answer of 15 is typically more attractive for this kind of problem because 15 is the smallest sales figure for which profit is not negative.

Example *A coffee shop collects $1.25 per cup of coffee. The fixed cost for running the shop is $30 per day, and the cost of the materials per cup of coffee is $0.63 . How many cups of coffee need to be sold per day in order to break even?*

Let x be the Number of cups of coffee sold in a day.
(Revenue per cup)(Number of cups) - (Fixed cost) - (Cost per cup)(Number of cups) = Profit.

The break-even point is the number of cups sold in order for the profit to be zero or just above zero. We need to solve the equation $1.25x - 30 - .63x = 0$.

$$\begin{aligned} 1.25x - 30 - .63x &= 0 \\ .62x - 30 &= 0 \\ +30 & \quad +30 \\ .62x &= 30 \\ \frac{.62x}{.62} &= \frac{30}{.62} \\ x &= 48.4 \end{aligned}$$

Note that we divided and rounded but kept at least one digit past the decimal point. Alwayskeep a decimal point in your result when at least one of your numbers uses a decimal point. For this problem, we answer the question in the only way that can make sense. One can only sell a non-negative integer number of cups of coffee. Answer in a complete, correct sentence whenever possible.

Answer: One must sell 49 cups of coffee to break even.

Example In a mixture of 360 bushels of grain, there is one-fifth as much corn as wheat. How many bushels are there of each?
There are two quantities to solve, bushels of wheat and bushels of corn. We know that

$$360 = Bushels\ of\ wheat + Bushels\ of\ corn$$

It is convenient to define a variable to be one of these unknowns then define the other unknown using the same variable.

"there is one-fifth as much corn as wheat" defines bushels of corn in terms of bushels of wheat, so if we let

$$x = bushels\,of\,wheat$$

We can define the bushels of corn in terms of x.

$$\frac{1}{5}x = bushels\,of\,corn$$

Then,

$$\begin{aligned}
360 &= Wheat + Corn \\
360 &= x + \frac{1}{5}x \\
&= \frac{5}{5}x + \frac{1}{5}x \\
360 &= \frac{6}{5}x \\
5(360) &= 5 \cdot \frac{6}{5}x \\
1800 &= 6x \\
\frac{1800}{6} &= \frac{6x}{6} \\
300 &= x
\end{aligned}$$

So, there are 300 bushels of wheat. Then

$$\begin{aligned}
bushels\,of\,corn &= \frac{1}{5}x \\
&= \frac{1}{5}(300) \\
&= 60
\end{aligned}$$

Thus, there are 60 bushels of corn.

Exercises Solve the problems using algebra.

1. A dealer sells tires for \$50 each. The tires cost the dealer \$35 each. The dealer's fixed costs are \$120 per day. How many tires must be sold per day to break even?
2. One number is four times another, and the sum of the two numbers is twenty. Represent the smaller number by x, then represent the the larger number. What are the two numbers?
3. A man started shopping with \$4195. After shopping for a while, he found that he had left four times as much as he had spent. How much did he spend?
4. The perimeter of a rectangle is 132 inches. the length is twice the width. What are the dimensions of the rectangle?
5. The sum of two numbers is 30, and the greater number exceeds the lesser number by 4. What are the two numbers?
6. A man travels 1120 miles by car and plane. He goes 450 miles further by plane than by car. how many miles did he travel by car, and how many miles did he travel by plane?
7. During an election, two candidates A and B had 653 votes in total, and A was beaten by 395 votes. How many votes did each receive?
8. A field is seven times as long as it is wide, and the distance around the field is 240 feet. What are the dimensions of the field?
9. Two farms differ by 250 acres, and the area of one is six times the area of the other. How many acres are in each?
10. The sum of the ages of a mother and a daughter is 32 years, and the age of the mother is seven times the age of the daughter. What is the age of each?
11. A man bought a hat, a pair of shoes, and a necktie for \$75. The hat cost four times as much as the necktie, and the boots cost five times as much as the necktie. What was the cost of each?
12. A field containing 45,000 square feet was divided into three lots so that the second lot was three times the first, and the third was twice the second. How large was each lot?
13. In a class of 35 students, there are 7 more men than women. how many are there of each?
14. A bar has fixed costs of \$140 per day. Each beverage sold for \$3 costs \$1 in supplies. What is the break-even sales for beverages?
15. A bookstore has fixed costs of \$210 per day. The store receives 23% of the price of each book sold. To the closest nickel, what is the dollar sales value of books necessary for the bookstore to break even?