2. Linear equations and inequalities

Rules of logic particular to mathematics apply for all equations and inequalities. There is a left side and a right side for each equation or inequality. The general method for solving is to do the same thing to both sides in order to obtain an equivalent equation or inequality. The goal is to solve for a variable, and this is accomplished when the variable is alone on one side and the variable does not appear on the other side.

1. If $A = B$ then $B = A$.
   This is the symetric property of equality. You can switch both sides of an equation.
   Example: $\begin{aligned} 2 &= \text{x} \\ \text{x} &= 2 \end{aligned}$
2. If A = B, then A + C = B + C.
   You can add the same thing to both sides of an equation.
   Example: $\begin{aligned} \text{x - 3} &= 5 \\ \text{x - 3 + 3} &= 5 + 3 \end{aligned}$
3. If A = B, then A - C = B - C.
   You can subtract the same thing from both sides.
   Example: $\begin{aligned} \text{x + 12} &= 7 \\ \text{x + 12 - 12} &= 7 - 12 \end{aligned}$
4. If A = B, then AC = BC.
   You can multiply both sides by the same thing, but this only works if C is not zero.
   Example: $\begin{aligned} \frac{2}{3}x &= 7 \\ \frac{3}{2}\cdot\frac{2}{3}x &= \frac{3}{2}\cdot 7 \end{aligned}$
5. If A = B, then $\frac{A}{C} = \frac{B}{C}$.
   You can divide both sides by the same thing, but C cannot be zero.
   Example: $\begin{aligned} \text{6x} &= 5 \\ \frac{6x}{6} &= \frac{5}{6} \end{aligned}$
6. Replace A(B + C) with AB + AC.
   You can use the distributive law.
   Example: $\begin{aligned} \text{4(x - 9)} &= 7 \\ 4\cdot x - 4\cdot 9 &= 7 \\ \text{4x - 36} &= 7 \end{aligned}$
7. Replace A + B with B + A, and replace AB with BA.
   You can use the commutative laws.
   Example: $\begin{aligned} \text{-7 + x} &= 3 \\ \text{x - 7} &= 3 \end{aligned}$
8. Replace $A\cdot 1$with A, and replace $1\cdot A$ with A.
   You can use the propeties of one.
   Example: $\begin{aligned} 1\cdot x &= 5 \\ x &= 5 \end{aligned}$
9. Replace A + 0 with A, and replace 0 + A with A.
   You can use the properties of zero.
   Example: $\begin{aligned} \text{x + 0} &= 0 + 7 \\ \text{x} &= 7 \end{aligned}$
10. Replace $\frac{A}{A}$with 1.
    You can use the cancelation property.
    Example: $\begin{aligned} \frac{3x}{3} &= \frac{12}{3} \\ \frac{3}{3}\cdot\frac{x}{1} &= \frac{3}{3}\cdot\frac{4}{1} \\ 1\cdot x &= 1\cdot 4 \end{aligned}$
11. Replace A + (-A) with 0.
    You can use the additive inverse property.
    Example: $\begin{aligned} \text{x + 4 + (-4)} &= \text{7 + (-4)} \\ \text{x + 0} &= 3 \end{aligned}$
12. Replace $\frac{A}{B}\cdot\frac{B}{A}$with 1.
    You can use the muliplicative inverse property.
    Example: $\begin{aligned} \frac{5}{7}\cdot\frac{7}{5}x &= \frac{5}{7}\cdot 21 \\ 1\cdot x &= 15 \end{aligned}$

The properties above may be used in any order, and any resulting equation is called an equivalent equation. Equivalent equations have the same solutions or, otherwise, contain the same information even if they do not look the same.

2.1 Introduction to Linear Equations

In solving an equation for a variable, you want to end up with something like $x =$ Everything Else. The strategy here is to use addition and subtraction first to get all variable terms on one side and all numbers on the other side. All like terms should be combined, variables with variables, and numbers with numbers. For example, x + x = 2x, 3x + 5x = 8x, and 4x - x = 3x. Then, divide or multiply to eliminate the coefficient of the variable.

Strategy for solving a simple linear equation in one variable

1. Add and subtract to get variables on one side, numbers on the other side.
2. Combine all like terms for each side.
3. Divide or multiply to cancel the coefficient of the variable.

Example Solve: $x - 8 = 4$.

$$\begin{array}{rcl} & \text{x - 8} & = 8 \\ \oplus & 8 & = 9 \\ \hline & \text{x - 8 + 8} & = 4 + 8 \\ & \text{x} & = 12 \end{array}$$

Example Solve: 3x = 12.

$$\begin{array}{rcl} \text{3x} & = & 12 \\ \frac{3x}{3} & = & \frac{12}{3} \\ \text{x} & = & 4 \end{array}$$

Example Solve 2 + 3x - 7 = 12 .

$$\begin{array}{rcl} & \text{2 + 3x - 7} & = 12 \\ & \text{3x - 5} & = 12 \\ \oplus & 5 & 5 \\ \hline & \text{3x} & = 17 \\ & \frac{3x}{3} & = \frac{17}{3} \\ & \text{x} & = \frac{19}{3} \end{array}$$

Example Solve $\frac{2}{3}x - 9 = 4$.

$$\begin{array}{rcl} & \frac{2}{3}x - 9 & = 4 \\ \oplus & 9 & 9 \\ \hline & \frac{2}{3}x & = 13 \\ & \frac{3}{2} \cdot \frac{2}{3}x & = \frac{3}{2} \cdot 13 \\ & \text{x} & = \frac{39}{2} \end{array}$$

Exercises Solve the following equations for the unknown variable.

1. $x + 6 = 3$
2. $-x = 9$
3. $x + 23 = 86$
4. $2x = -12$
5. $3 - x = 14$
6. $x + 3 = -1$
7. $6x = 90$
8. $-3x = 42$
9. $\frac{x}{5} = -8$
10. $\frac{3}{2}x = 18$
11. $x + 6 = 20$
12. $4x = -28$
13. $x + 9 = 30$
14. $5x = -40$
15. $x - 2 = -4$
16. $x - 10 = 3$
17. $4x = -64$
18. $-3x = 72$
19. $8 - x = 12$
20. $23 + x = 4$