1.5 Variable Expressions

A variable is a letter which stands for a number. Everything you can do with numbers you can do with variables, and vice-versa. There are attempts to classify variables as unknowns, as solvable quantities, as constants, and such not, yet it is easiest and correct to simply treat variables as numbers which may or may not be known.

Any letter can be used as a variable. Popular letters for algebra are lower case Latin letters like a, b, c, x, y, and z. There are some letters which are typically reserved for special use, like i and e which usually stand for particular numbers. There are other letters, like o, which might be confused with zero (0) in many circulmstances and are thus avoided. The most popular variable letter, by far, is x.

The popularity of the letter x appears to be longstanding as "x" was once a shortcut for "thing" as used in ancient algebra texts like Al-Jabr.

Not knowing what number a variable actually stands for does not prevent us from applying various rules of arithmetic. All rules of arithmetic for numbers do apply to variables.

Exponents mean repeated multiplication, as $2^3 = 2 \cdot 2 \cdot 2 = 8$. We cannot finish up an expression like x^3 except to note that $x^3 = xxx$, three x's multiplied together. In fact, the exponent was not used as we use it until Rene Descartes' time (1637) when Descartes used positive integer exponents of 3 and more to write quantities like x^3, x^4, and so on. Descartes still wrote x^2 as xx, however.

Multiplication means repeated addition. This is not really something to think about when we run across an expression like $2 \cdot 3$. We might know that $2 \cdot 3 = 3 + 3$ (also 2 + 2 + 2) , yet we have already moved on to simply knowing multiplication facts. However, the definition of multiplication as repeated addition must be remembered to handle things like $x + x$. $x + x = 2x$, $x + x + x = 3x$, $x + x + x + x = 4x$, and so on.

In addition variable expressions such as $10x + 13x$, we can think of there being 10 x's and 13 x's being added up, so 10 + 13 = 23, and $10x + 13x = 23x$. If you recall the expression "comparing apples to oranges", the point of this saying is that apples are different from oranges. Four apples and seven oranges added to twenty apples and two oranges is (4 + 20) apples and (7 + 2) oranges or 24 apples and 9 oranges. Similarly,

$$\begin{aligned}(4x+7)+(20x+2) &= (4+20)x+(7+2) \\ &= 24x+9\end{aligned}$$

In short, we combine like terms, add numbers to numbers, x's to x's, and so forth.

Example Simplify the expression $3x+10-x+18$.

$$\begin{aligned}3x+10-x+18 &= 3x-x+10-18 \\ &= 2x-8\end{aligned}$$

Examples Simplify the following.

1. $3(5x-8)$
   Apply the distributive law first.

$$\begin{aligned}3(5x-8) &= 3\cdot 5x - 3\cdot 8 \\ &= 15x-24\end{aligned}$$

2. $\frac{2}{3}\cdot 24x$
   Multiply an expression times the numerator of a fraction.

$$\begin{aligned}\frac{2}{3}\cdot 24x &= \frac{2\cdot 24x}{3} \\ &= \frac{48x}{3} \\ &= \frac{3\cdot 16x}{3} = 16x\end{aligned}$$

3. $4x-(5x-7)$
   The distributive law is used here, but use it carefully. In subtracting $(5x-7)$, you subtract $5x$, then subtracting a negative as in -7 results in adding the seven. You can think of this like

$$-(-7) = 7$$

Then

$$\begin{aligned}4x-(5x-7) &= 4x-5x-(-7) \\ &= 4x-5x+7 \\ &= -x+7\end{aligned}$$

This result, $-x+7$ is fine as-is. However, it is sometimes written as $7-x$. You should know that these forms are equal and both correct.

Example Evaluate the expression $5y + \dfrac{x^2 - y^3}{xy+1}$ when x= -2 and y = 3.

$$\begin{aligned} 5y + \frac{x^2 - y^3}{xy} &= 5(3) + \frac{(-2)^2 - (3)^3}{(-2)(3)+1} \\ &= 15 + \frac{-8-27}{-6+1} \\ &= 15 + \frac{-35}{-5} \\ &= 15 + 7 \\ &= 22 \end{aligned}$$

Exercises

Simplify the following.

1. 12x + x - 6x
2. $\frac{5}{3} \cdot (30x)$
3. $x + 6 + 2x + 8 + 5x + 1 + x$
4. $-5(x^2 + 7x - 2)$
5. $x - (2x - 10)$
6. 7x - 12x + x
7. $\frac{3}{4} \cdot (60x)$
8. $-8(3x^2 - 2x - 11)$
9. $8x - (2x - 10)$
10. 4x + x - 10x
11. $\frac{2}{5} \cdot (40x)$
12. $-3(2x^2 - x + 8)$
13. $5x - (3x - 7)$
14. Evaluate $a^2 - 5b$ where a = -2 and b = 6 .
15. Evaluate $3a - b^2$ where a = -5 and b = 4 .
16. Evaluate $a^2 - 3b + a$ where a = - 3 and b = 5 .
17. Evaluate $\dfrac{a+5b}{a-b}$ where a = 5 and b = -2