# 1.3 Integer Exponents, Percents, Square Roots, and Decimals

## Exponents

Positive integer exponents are a shortcut. $3^2$means $3 \cdot 3$, $4^3$ means $4 \cdot 4 \cdot 4$, $7^5$means $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$and so on. If the result is fairly small, it is usually computed. $3^3 = 27$. When the result is large, the exponent form is often preserved as in $10^{23}$.

In general

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{\text{n times}}$$

However, there are a few special cases to point out.

$$\begin{aligned} a^1 &= a \\ a^0 &= 1 \end{aligned}$$

There is some special attention required for raising negatives to powers. The meaning of $-3^2$is $-3{\cdot}3 = -9$. If you really want to raise -3 to the second power, you must use parenthesis as in $(-3)^2 = (-3)(-3) = 9$.

## Percents

Percent means "hundredths" or "per hundred". This is the cent in one dollar. One hundred Percent (100%) is 100 hundredths or $\frac{100}{100} = 1$. One percent (1%) is one hundredth $= \frac{1}{100} = .01$.

$$\begin{aligned} 100.\% &= 1. \\ 1.\% &= .01 \end{aligned}$$

The decimal place is not usually included if the percent is an integer like 100% or 44%, but it is shown because it is easiest to think of moving decimal places when converted back and forth between decimals and percents.

**Example** Convert 6.5% to a decimal.
$6.5\% = .065$

**Example** Convert 1.25 to percent.
$1.25 = 125.\%$

To take a percent of something is to take parts of a hundred. "23% of 1000" means "take 23 of every 100 parts" which is most easily accomplished by multiplying by 23/100.

$$\begin{aligned} 23\%\, of\, 1000 &= \frac{23}{100} \cdot 1000 \\ &= \frac{23 \cdot 1000}{100} = \frac{230 \cdot 100}{100} = 230 \end{aligned}$$

Usually, percents are calculated by converting the percentage to a decimal then multiplying, and this is equivalent to using fractions but can be easily done on a calculator.

$$23\%\, of\, 1000 = .23(1000) = 230.$$

## Decimals

When you see a decimal as in 23.5, you are being told that there are three significant digits of precision and, perhaps, that 23.5 was obtained by rounding to the closest tenth. Thus, when you see decimals in an expression, you are given license to use decimals for the entire expression, and, further, you should express your answer as a decimal. Even an integer with a decimal point, like 115., is telling you that 115 is an approximation. This is typical of all measured quantities like inches, kilograms, and seconds.

## Conversion from Fractions to Decimals

When division of an integer by a non-zero integer is carried out, the result is a terminating or repeating decimal.

**Example** Convert $\frac{3}{8}$to a decimal.
$3/8 = .375$, a terminating decimal.

**Example** Convert $\frac{5}{3}$to a decimal.
$5/3 = 1.66666\cdots = 1.\overline{6}$. The overbar means that the 6 digit repeats forever.

## Rounding

When rounding to a particular place, if the following digit is 0-4, drop everything after the place. If the following digit is 5-9, add 1 to the digit in place.

**Example** Convert $\frac{3}{7}$to a decimal and round to the closest hundredth (to two decimal places.)
$3/7 = 0.428571428571\cdots = 0.\overline{428571}$. Rounded to the closest hundredth, this is 0.43 because the digit after the 2 in the hundredth's place is 8, so 1 is added to the 2 in .42 or .42 + .01 = .43 .

**Example** Convert $\frac{1}{3}$to a decimal rounded to the closest thousandth (to 3 decimal places.)
$1/3 = .3333\cdots = .\overline{3}$ . Rounded to the closest thousandth this is .333 because the digit following the 3 in the thoiusandth place is 3, so everything following is dropped.

When you are given decimal numbers in a problem, the number of significant digits tell you how precise the number is to begin with. The number of significant digits is the the number of nonzero digits after the first nonzero digit.

13.15 has 4 significant digits.

0.0031 has 2 significant digits. The leading zeros do not count in determining the number of significant digits..

870,000 has two significant digits. Nothing is perfect here, and it is actually unclear whether there are 2, 3, 4, 5, or 6 significant digits. We assume that there are 2 significant digits, however. If there really were more, we would use scientific notation to make this point clear.

The general rule for rounding the result is that the final answer should not have more significant digits than any of the numbers used to get the final answer.

**Example** 435 cartons of strawberries were bought. Each carton weighs 1.7 lb. How many pounds of strawberries were bought?

$$1.7(435) = 739.5$$

The result from the calculation is 739.6 lb. However, there are only 2 digits of precision in the weight of each carton, so we round to 2 significant digits. The second digit is 3, and the digit following is a 9, so we round the second digit up to 4. The answer is 740 lb of strawberries.

## Square Roots

The square root of a number x is a number which when squared equals x. There are two such square roots of every number except for zero. The square roots of 9 are 3 and -3 because

$$\begin{aligned} 3^2 &= 9 \\ (-3)^2 &= 9 \end{aligned}$$

The principal square root of a number is the positive square root of the number, and the principal square root of a number x is indicated by putting x inside a radical or square root symbol.

$$\sqrt{x}$$

$\sqrt{4} = 2$ because $2^2 = 4$, $\sqrt{9} = 3$ because $3^2 = 9$, and so forth.

In fact, here are the common perfect squares you will be taking square roots of: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, and 196. This list can go on infinitely, yet these are the perfect squares you have likely seen from memorizing multiplication tables. For example, you already know that $12 \cdot 12 = 144$, and this means that $12^2 = 144$, so $\sqrt{144} = 12$.

On the other hand, most numbers are not perfect squares. $\sqrt{5}$ cannot be simplified further because 5 is not a perfect square. $\sqrt{5}$can be approximated using a calculator, and it is 2.236 to three decimal places or four significant digits. Yet, square roots of numbers which are not perfect squares are irrational which means that they cannot be exactly expressed as a fraction.

It is sometimes useful, however, to take the square root of a factor of a number when the factor is a perfect square. For example, 12 is not a perfect square, but 12= 4·3, and 4 is a perfect square. We use this rule for taking square roots of products.

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

**Example** Simplify $\sqrt{12}$.
$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$.

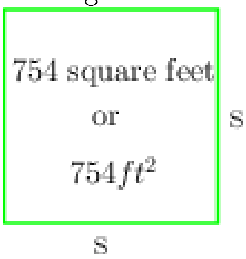
**Example** Simplify $\sqrt{98}$.
$\sqrt{98} = \sqrt{49 \cdot 2} = \sqrt{49}\sqrt{2} = 7\sqrt{2}$.

**Example** Simplify $\sqrt{30}$.

This is already simplified because 30 does not contain a perfect square over 1 as a factor.

In applied problems where numbers are approximate, you will want to use decimal approximations to square roots.

**Example** What are the dimensions of a square room which has 754 square feet of flooring?




The area of the room is $754ft^2$, and the area formula for a square is $A = s^2$ where s is the length of the sides of the square. Then

$$\begin{aligned} s^2 &= 754 \\ s &= \sqrt{754} \\ &\approx 27.4590604355 \end{aligned}$$

This result of 27.4590604355 was read from a scientific calculator, and there are too many digits because there are only 3 digits of accuracy in the room area figure of 754. Thus, we round the result to 27.5, and our answer is 27.5 ft per side for the dimensions of the square room.

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## Exercises

Simplify to an integer.

1. $7^2$
2. $(-4)^3$
3. $\left(\frac{-2}{3}\right)^3$
4. $12^0$
5. $3^2 + 3^1 + 3^0$

Calculate.

6. 20% of 140
7. 4.05( 5.1 )
8. 1.48(-2.5)
9. Write as a decimal. $\frac{11}{8}$
10. Write as a fraction. 43%
11. Write $\frac{5}{16}$ as a decimal.
12. Write 42% as a decimal.
13. Write as a percent. $\frac{7}{5}$
14. Write $\frac{5}{8}$ as a decimal.
15. Write 12.5 % as a decimal.
16. Write $\frac{3}{8}$ as a decimal.
17. Write 35% as a decimal.

Simplify.

18. $5^3$
19. $(-2)^3$
20. $\sqrt{36}$
21. $\sqrt{75}$
22. $\sqrt{36}$
23. $\sqrt{200}$
24. $\sqrt{121}$
25. $\sqrt{300}$
26. $\sqrt{81}$
27. $\sqrt{50}$