1. Real Numbers and Variables

1.1 Integers

Integers

The integers are the positive and negative whole numbers. There are infinitely many integers as indicated by the elipses "$\cdots$" which mean "and so on and so forth".
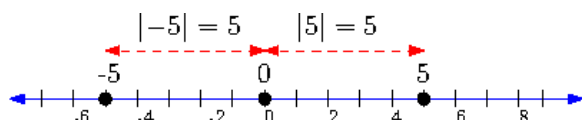
$$\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$$

Subtraction and Adding Integers

Subtraction is the same as adding the opposite. 7-4 is the same as 7 + (-4).

When adding numbers which all have a negative sign, add the numbers as if there were no signs, then prefix the sum with the negative sign.

Example -2 - 5 + (-10) = (-2) + (-5) + (-10) = -(2+5+10) = -17

The absolute value of a number is the distance from the number to zero.

When finding a difference, or adding a negative and a positive number, subtract the smaller absolute value from the larger absolute value, then put the sign of the parger number in front of the result.

Example 20 - 7 = 13

Example 14 - 31 = -(31-14) = -(17) = -17

Example 16 + (-21) = -(21-16) = -(5) = -5

For subtraction, it sometimes helps to think about money in, say, a checking account. Deposits are positive. Checks or withdrawals are negative. Add all the deposits together (positives), add all the negatives together (withdrawals), then subtract the smaller result from the larger and put on a negative if the withdrawals were larger than the deposits.

When adding numbers together where are some are positive and some are negative, add the positive numbers together, add the negative numbers together, then find the difference.

Example 4 - 8 + 20 + (-2) - 12 +10 + 30 - 50 = (4 + 20 + 10 + 30) - (8 + 2 + 12 + 50) = 64 - (72) = -(72-64) = -8

Multiplying Integers

The result of multiplying two or more numbers is the product. The product of 3 and 4 is 12; $3 \cdot 4 = 12$. Multiplication can be indicated by a dot as in $5 \cdot 12 = 60$, and this is usual when multiplying positive integers. Putting a number next to another number inside parenthesis also means multiplication. $5(8) = 40$. Parenthesis are sometimes necessary when multiplying negative numbers as in $6(-7) = -42$, but this is the same as $-7 \cdot 6 = -42$.

The rule on signs when multiplying is that a positive times a positive is positive, a negative times a negative is positive, but a negative times a positive is negative.

Example $(9)(7) = 63$

Example $(-9)(7) = -63$

Example $(9)(-7) = -63$

Example $(-9)(-7) = 63$

When numbers are multiplied together, you can determine the sign of the product by counting the number of negative numbers. If there is an even number of negative numbers, the product is positive. If there is an odd number of negative numbers, the product is negative. It is as if each pair of negative numbers cancels eachother out, and this is the correct view.

Example $(-1)(-2)(-5) = -10$
The product is negative because there are 3 negatives, and 3 is odd.

Example $(-1)(-2)(-3)(-5) = 30$
The product is positive because there are 4 negatives, and 4 is even.

Dividing Integers (evenly)

When the numerator is a multiple of the denominator, the result (quotient) is an integer. Twelve divided by three is four because four times three is twelve. Twelve is the dividend, three is the divisor, and four is the quotient.

$$\frac{Dividend}{Divisor} = Quotient$$
$$\frac{12}{3} = 4$$

Determining the sign of the quotient is the same as that for multiplication.

Example $\dfrac{-14}{2} = -7$

Example $\dfrac{36}{-9} = -4$

Example $\dfrac{-20}{-4} = 5$

Exercises

Perform the operations and simplify to a single integer.

1. $4 - 7 - 23$

2. $7 - 10 - (-12)$

3. $-14 + (-5) + 91 + (-100)$

4. $(-6)(-5)$

5. $(-24) \div (-2)$

6. $-\dfrac{-120}{-5}$

7. $(-3)(-8)$

8. $\dfrac{12}{-3}$

9. $\dfrac{-42}{-3}$

10. $4 + (-8) - 7 - (-3)$

11. $|4|$

12. $|-345|$

13. $(-1)(-2)(-3)(-4)$

14. $(-5)7(-2)$

15. $2 \cdot 3 \cdot 7$

16. $(-9)7$

17. $9(-11)(-243)(-613)(-99)0$

18. $3 + 9$

19. $10 - 23$

Answer these questions.

20. What is the opposite of -12 ?

21. What is the opposite of -104 ?

Perform the operations, then simplify.

22. 32 - 61

23. - 14 + 23 + (-11)

24. 29 - (-6) - 18

25. $240 \div (-12)$

26. $64 \div (-8)$

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